JUNIOR ARITHMETIC

BORCHARDT

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JUNIOR ARITHMETIC

BY

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JUNIOR ARITHMETIC

This Book is issued in four ways—

1. Complete in One Volume, with Answers, 2s.
2. ,, ,, without Answers, 1s. 6d.
3. The Examples separately, with Answers, 1s. 6d.
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PREFACE

THIS volume is based on the author's *Arithmetical Types and Examples*, but at the same time contains much new matter. Greater prominence has been given to the more elementary parts of the subject, while the more difficult sections have been omitted.

Fully worked out model solutions have been supplied, and copious notes on theory inserted throughout the book.

The following are some of the distinguishing features :—

1. The new method of multiplication is used exclusively.
2. *Decimals* and the Metric System are placed before Vulgar Fractions.
3. In both Multiplication and Division of Decimals it is so arranged that there is one digit only to the left of the decimal point in the multiplier and divisor. (See pages 50, 52.)
4. It is not necessary in any examples to use the fact that $5\frac{1}{2}$ yards = 1 pole
   or $30\frac{1}{4}$ sq. yards = 1 sq. pole.
5. The importance of *Rough Checks* is insisted on throughout the volume.

6. Simple examples on Areas, Volumes, and Unita
Method are introduced early.

7. Recurring Decimals have been omitted exc
in so far as they are necessary for the red
tion of a vulgar fraction to a decimal a
vice versâ.

8. Illustrations are given of the Geometri
Method for the extraction of Square Roo
and a section on the *Application of Gra*
is inserted.

9. There are important sections on *Contrac*
Work of all kinds, *Mensuration* of a ci
and cylinder, and practical questions
Percentages.

10. In the later sets of examples, money answ
are usually required to the nearest penny or
and the application of Tables to Compou
Interest is shown. Prominence is also give
the connection between problems on Pres
Worth and Discount, and those on Ordin
Interest.

11. A set of *Harder Miscellaneous Problems* (ta
from recent Examination Papers) has b
placed immediately before the 55 *Test Pap*

It is hoped that the book will prove useful for
Cambridge and Oxford Locals, London Matriculat
and other examinations.

CHELTENHAM COLLEGE,
April 1906.

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JUNIOR ARITHMETIC

TABLES

Time.

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

1 gross = 144
1 score = 20
1 dozen = 12

Money.

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound (£)

1 florin = 2s.
1 half-crown = 2s. 6d.
1 crown = 5s.
1 half-sovereign = 10s.
1 guinea = 21s.

∴ 10 *florins* = £1
8 *half-crowns* = £1
4 *crowns* = £1

1 franc = 100 centimes
1 dollar = 100 cents

Avoirdupois Weight.

16 drams = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
28 pounds = 1 quarter (qr.)
4 quarters = 1 hundredweight (cwt.)
20 hundredweight = 1 ton

∴ 112 *lbs.* = 1 *cwt.*
2240 *lbs.* = 1 *ton*
14 *lbs.* = 1 *stone*

Troy Weight.

24 grains = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

1 *lb.* (*Avoir.*) = 7000 *grs.*
1 *lb.* (*Troy*) = 5760 *grs.*

Length.

12 inches = 1 foot (ft.)
3 feet = 1 yard (yd.)
220 yards, 10 chains, 40 poles or rods = 1 furlong (fur.)
8 furlongs = 1 mile (m.)

$5\frac{1}{2}$ *yards* = 1 *pole*
22 *yards* = 1 *chain* = 100 *links*
1760 *yards* = 1 *mile*

Area.

144 sq. inches = 1 sq. foot
9 sq. feet = 1 sq. yard
1210 sq. yards, 40 perches, sq. poles or sq. rods = 1 rood (rd.)
4 roods = 1 acre (ac.)

---

640 *acres* = 1 *sq. mile*
4840 *sq. yards* = 1 *acre*
$30\frac{1}{4}$ *sq. yards* = 1 *sq. pole*

Volume.

1728 cu. inches = 1 cu. foot
27 cu. feet = 1 cu. yard

Capacity.

2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gall.)
2 gallons = 1 peck (pk.)
4 pecks = 1 bushel (bush.)
8 bushels = 1 quarter (qr.)

METRIC SYSTEM

[*The sub-divisions have Latin prefixes, and the multiples Greek prefixes.*]

Length.

10 *milli*metres (mm.) = 1 *centi*metre (cm.)
10 *centi*metres = 1 *deci*metre (dm.)
10 *deci*metres = 1 metre (m.)
10 metres = 1 *deka*metre (Dm.)
10 *deka*metres = 1 *hecto*metre (Hm.)
10 *hecto*metres = 1 *kilo*metre (Km.)

∴ 1 kilometre = 1000 metres
1 hectometre = 100 metres, etc.

Area.

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 sq. m.
100 sq. m. = 1 sq. Dm.
100 sq. Dm. = 1 sq. Hm.
100 sq. Hm. = 1 sq. Km.

1 *are* = 1 sq. Dm.

Volume.

1000 cu. mm. = 1 cu. cm.
1000 cu. cm. = 1 cu. dm. or 1 *litre*
1000 cu. dm. = 1 cu. m.
1000 cu. m. = 1 cu. Dm.
1000 cu. Dm. = 1 cu. Hm.
1000 cu. Hm. = 1 cu. Km.

∴ 1 *litre* = 1 cu. dm. = 1000 cu. cm.

Mass.

The unit, the mass of 1 *cu. cm. of pure water at* 4° C, *is named gram.*

10 *milli*grams (mg.) = 1 *centi*gram (cg.)
10 *centi*grams = 1 *deci*gram (dg.)
10 *deci*grams = 1 gram (g.)
10 grams = 1 *deka*gram (Dg.)
10 *deka*grams = 1 *hecto*gram (Hg.)
10 *hecto*grams = 1 *kilo*gram (Kg.)

ADDITION

[In adding 2, 3, 9, 11, say 2, 5, 14, 25, and not 2 and 3 make 5, and 9 make 14, and 11 make 25.]

Exercise I. (*Oral.*)

State in words :—

1. 3,251.
2. 827.
3. 5,033.
4. 7,205.
5. 10,205.
6. 82,709.
7. 10,005.
8. 12,309.
9. 143,071.
10. 3,007,895.

What is the value of :—

11. $2+5+8+9$.
12. $4+22+5+7$.
13. $8+2+5+29$.
14. $10+11+12+13$.
15. $14+16+18+20$.
16. $15+20+25+30$.
17. $5+12+19+26$.
18. $8+16+24+32$.
19. $10+15+20+50$.
20. $7+9+17+15$.
21. $8+7+9+12$.
22. $14+15+17+18$.
23. $2+7+12+22$.
24. $5+15+20+25$.
25. $4+8+12+16$.
26. $3+9+27+81$.
27. $5+25+125$.
28. $5+9+13+17$.
29. $11+18+25+32$.
30. $13+19+25+31$?

Exercise II.

Find the sum of :—

1. 32, 907, 1172, 1307.
2. 45, 729, 1103, 4560.
3. 82, 970, 305, 3009.
4. 1001, 101, 111, 1010.
5. 105, 7831, 87459, 83801, 1234, 421, 86, 125.
6. 5432, 983, 598, 1109, 768, 543, 890, 674.
7. 758, 22727, 4375, 4375, 85, 68, 9127, 43761.
8. 294, 3467, 8987, 68958, 93486, 29345, 641.

Find the value of :—

9. $12+86+576+35+249+768$.
10. $11+321+497+231+112+1007$.
11. $15+17+821+970+1102$.
12. $850+31+92+1178+35$.
13. $420+479+1108+723+1728$.
14. $1+8+27+64+125+216+343+512$.
15. $25+36+49+64+81+100+121+144+169$.
16. $78+87+341+143+721+127+1428$.

Add the following columns :—

17.	3247	**21.**	52761	**25.**	81506
	123		8245		2051
	8205		723		820715
	72		82915		42035
	41324		7623		7206
	8725		84297		82911
18.	1307	**22.**	8241	**26.**	40052
	214		762		8173
	896		9824		902051
	54281		5102		89999
	7498		82307		23478
	2342		82		9269
19.	1823	**23.**	82751	**27.**	62407
	414		1206		8102
	728		34215		9925
	94562		87018		803409
	8785		20501		2892
	902		7021		777
20.	23	**24.**	42607	**28.**	34276
	10234		8221		1234
	8275		89052		9876
	106		4217		5432
	82415		826		102345
	98723		914827		78562

9. 187915	**30.** 876234	**31.** 824561	**32.** 147852
24627	7892	2403	4290
814279	40035	14276	82705
82491	726	634825	9899
7205	81429	7892	40708
10025	100481	46785	241827

SUBTRACTION

Ex. *Subtract* 42183 *from* 52374.

52374
42183
10191 Answer.

[Obtain the figures in the bottom line by ascertaining how much must be added to the second line to obtain the first line. The mental process should be as follows :—

3 and 1 make 4 ; 8 and 9 make 17 ; carry 1 ; 2 and 1 make 3 ; 2 and 0 make 2 ; 4 and 1 make 5.]

Exercise III. (*Oral.*)

What is the remainder when—

1. 3 is subtracted from 8, 12, 17, 19, 22, 28, 31, 43.
2. 5 „ „ 17, 19, 25, 32, 37, 45, 52, 59.
3. 6 „ „ 22, 29, 33, 38, 42, 57, 63, 81.
4. 9 „ „ 32, 35, 41, 52, 63, 74, 77, 83, 92?

Find the values respectively of :—

5. 72 – 11, 83 – 11, 42 – 11, 79 – 11, 82 – 11, 99 – 11.
6. 87 – 12, 92 – 12, 48 – 12, 77 – 12, 93 – 12, 44 – 12.
7. 18 – 9, 72 – 8, 43 – 7, 59 – 13, 72 – 18, 45 – 14.
8. 8 + 7 – 12, 4 + 9 – 3, 7 – 2 + 8, 9 + 12 – 4, 8 – 7 + 11.
9. 4 – 3 + 9, 15 – 7 + 21, 14 + 7 – 9, 23 – 18 + 9, 25 – 11 + 12.
10. 18 – 7 + 1, 19 – 3 + 21, 22 – 17 + 11, 44 – 22 + 33, 9 – 12 + 87.

Exercise IIIa.

Subtract :—

1. 4257 from 5392.
2. 5341 from 12345.
3. 3001 from 7281.
4. 42567 from 345921.
5. 72341 from 94002.
6. 82759 from 523410.
7. 823001 from 942000.
8. 72401 from 98420.
9. 23475 from 34257.
10. 82341 from 94561.

Subtraction

Find the excess of :—

1. 724151 over 482310.
2. 834001 over 94756.
3. 97245 over 89725.
4. 14357 over 9287.
5. 28234 over 9460.
16. 582789 over 99999.
17. 425680 over 82759.
18. 10023 over 3001.
19. 50090 over 8271.
20. 72456 over 3425.

1. What number must be added to 8275 to give 10244 ?
2. Find the sum of 2047 and 1025 and then subtract it from 4000.
3. What must be added to the difference between 2175 and 1029 to produce 2505 ?
4. Add together two million eight hundred and fifty, five thousand four hundred and seventy ; subtract the sum from three million four hundred.
5. What number must be subtracted from 5237 to produce 2342 ?
6. To the difference between 42192 and 32415 add the difference between 7202 and 3049.
7. The populations of the different parishes of a town are 3245, 12793, 8241, 7239, and 4207 respectively ; by how much does the whole town's population fall short of 40,000 ?
8. Find the sum and difference of 32497 and 14208.
9. The sum of two numbers is 8249, and one of them is 3217 ; what is the other ?
0. The difference between two numbers is 327 and the greater is 8299 ; find the other.

MULTIPLICATION

Exercise IV. (*Oral.*)

Multiply :—

1. 17, 15, 22, 24 by 5.
2. 18, 22, 36, 45 by 20.
3. 14, 15, 20, 25 by 120.
4. 82, 34, 45, 52 by 50.
5. 78, 82, 35, 47 by 9.
6. 85, 47, 62, 57 by 11.
7. 32, 45, 62, 49 by 60.
8. 400, 320, 325, 415 by 30.
9. 82, 51, 29, 72 by 500.
10. 58, 97, 65, 42 by 11.

What are factors each less than 12 of :—

11. 36, 48, 56, 32.
12. 63, 77, 84, 105.
13. 280, 112, 96, 72.
14. 120, 121, 198, 112.
15. 126, 240, 147, 224.
16. 42, 160, 343, 216.

Ex. *Multiply* 3578 *by* 205.

$$\begin{array}{r} 3578 \\ 205 \\ \hline 7156 \\ 17890 \\ \hline 733490 \end{array}$$ Answer.

[*Rough Check:* $4000 \times 200 = 800{,}000$.]

[Always start multiplying by the left-hand digit of the multiplier. In multiplying the 8 by the 2, it should be remembered that we are multiplying 8 *units* by 2 *hundreds*, and thus the 6 obtained must be placed in the hundreds column, *i.e., under the left-hand figure of the multiplier.*]

To multiply by **99,** multiply by 100 and subtract the original number.

" " **999,** " 1000 " " "

Exercise V.

Multiply :—

1. 792 by 5, 9, 8.
2. 210 by 12, 13, 7.
3. 78064 by 8, 9, 11.
4. 12259 by 25, 15, 13.
5. 35906 by 480, 37, 23.
6. 173 by 52. 49, 81.
7. 671 by 134, 87, 31.
8. 2171 by 398, 123, 317.
9. 2133 by 416, 523.
10. 7613 by 1006, 925.

Multiply each of the numbers in the first column by each of the numbers the second column :—

11.	324.	(*a.*)	57.
12.	45.	(*b.*)	107.
13.	707.	(*c.*)	824.
14.	1823.	(*d.*)	7851.
15.	875.	(*e.*)	8026.
16.	8032.	(*f.*)	1247.

Multiply :—

17. 5275, 423, 8752, 5971 and 82301 by 99.

18. 3250, 502, 12034, 13003 and 5290 by 999.

19. A certain house contains 16 rooms. How many panes of glass will be required, if each room contains 2 windows and each window 12 panes of glass?

20. A draper sells on the average 37 reels of cotton per day. How many will he sell in 40 weeks (Sundays excepted)?

21. In building a viaduct of one hundred and one arches, ninety thousand and nine bricks are used for each arch. How many bricks are used altogether?

22. If a cyclist goes on the average 51 miles per day, how far will he go in 7 weeks?

23. If 57,692 parcels are posted every day, how many are posted in a year of 365 days?

DIVISION

Ex. 1. *Divide* 14964 *by* 12.

$$12\overline{)14964}\quad 1247$$

[*Check:* $1247 \times 12 = 14964$.]

Ex. 2. *Divide* 89346 *by* 140 *by short division.*

4 |	89346 units	
5 |	22336 units of 4	and 2 units
7 |	4467 units of 20	and 1 unit of 4
	638 units of 140	and 1 unit of 20
	638 and rem. 26.	Answer.

[*Remainder* $= 2 + 4 + 20 = 26$.]

[*Rough Check:* $90000 \div 140 = 643$ (approx.).]

Ex. 3. *Divide* 75239 *by* 251.

299 and rem. 190 Answer.

$$\begin{array}{r|l} 251 & 75239 \\ & 502 \\ \hline & 2509 \\ & 2259 \\ \hline & 2449 \\ & 2259 \\ \hline & 190 \end{array}$$

[*Rough Check:* $75000 \div 250 = 300$.]

The *Italian Method*, in which the alternate lines 502, 2259, 2259 are not written down, is not recommended for beginners.

To multiply by **5**, multiply by 10 and divide by 2.

" " **25**, " 100 " " 4.

" " **125**, " 1000 " " 8.

EXERCISE VI. (*Oral.*)

Divide :—

1. 27, 54, 57, 69, 72 by 3.
2. 16, 72, 76, 224, 332 by 4.
3. 25, 75, 95, 115, 145 by 5.
4. 56, 70, 91, 105, 574 by 7.
5. 96, 88, 56, 152, 144 by 8.
6. 108, 99, 117, 162, 891 by 9.
7. 121, 154, 198, 242, 275 by 11.
8. 144, 168, 180, 72, 228 by 12.

Multiply :—

9. 217, 418, 547, 189 by 5.
10. 14, 25, 62, 74 by 25.
11. 43, 72, 81, 93 by 125.
12. 81, 89, 77, 96 by 250.

Exercise VIa.

Divide by short division, checking the results by multiplication in Nos. 1 to 5 :—

1. 440, 520 and 3840 by 40.
2. 648, 792 and 1620 by 36.
3. 972, 2268 and 4050 by 54.
4. 1932, 2436 and 5880 by 84.
5. 5616, 6804 and 9828 by 108.
6. 872, 898 and 1024 by 55.
7. 892, 939 and 4278 by 140.
8. 5271, 8234 and 9021 by 99.
9. 5231, 7849 and 10234 by 144.
10. 3245, 8277 and 92301 by 245.
11. 3897, 7299 and 102307 by 315.
12. 82456, 998271 and 923007 by 132.

Divide by long division :—

13. 444565, 1265880 and 69111 by 137.
14. 896961, 1483541 and 4222532 by 211.
15. 1341019, 1572271 and 1586796 by 191.
16. 66056, 80487 and 813474 by 271.
17. 1084839, 361513 and 1122095 by 359.
18. 974487, 1705208 and 1437429 by 487.
19. 2449245, 3194455 and 207828 by 607.
20. 241769, 4106439 and 4418101 by 709.

Multiply :—

21. 327, 519, 8274, 4197 by 25.
22. 312, 427, 9271, 8925 by 125.
23. 83, 415, 729, 8203 by 175.

MISCELLANEOUS EXAMPLES

Exercise VII.

1. Amongst an audience of 3000 people, 1897 were men, 390 women, and 257 boys ; how many girls were there ?
2. The sum of two numbers is 4313 and the difference 2169 ; what are the numbers ?

3. Divide 34797 by 252, using short division.
4. If £3570 is divided as prize money amongst 42 seamen, how much will each receive?
5. How many apples will there be in 249 boxes if each box contains 396?
6. Find the number of letters on the page of a book, if each page contains 42 lines and each line 45 letters. If the book contains 375 pages, how many letters will there be in the book?
7. Find the value of 315 horses at £23 each.
8. Calculate the cost of 8520 tons of coal at 15 shillings per ton. Give the answer in shillings.
9. Show that the sum of 81459 and 54306 is equal to 5 times their difference.
10. Divide the difference between one hundred and seven thousand and ten, and twenty million, ten thousand, one hundred and one, by 25.
11. If seven apples cost threepence, what will be the cost in pence of 63 apples?
12. Multiply the sum of 3249 and 2467 by 327.
13. A certain fleet consists of 52 ships, and each ship on the average contains 527 men. How many men are there in the fleet?
14. The sum of two numbers is 5000005. The smaller number is one million and three. Determine their product.
15. How many times must 72 be added to 845 to give 1493?
16. A man buys 324 sheep at 32 shillings each, and 73 at 37 shillings each, and then sells the whole at 36 shillings each. How many shillings does he gain or lose?
17. If each of 25 people gathers apples at the rate of 12 per minute, how many apples will they gather in 35 minutes?
18. Multiply the difference between twenty-two thousand and six and nineteen thousand seven hundred by twenty-three.
19. What will be the result if 82976 is written down 11 times and added up?
20. What number multiplied by 28 will give the same result as 91 multiplied by 64?
21. Find the sum, difference, product, and quotient of 5544 and 504.
22. To the difference between 3417 and 2069 add the excess of 4217 over 3921. Multiply the result by 55.

23. What is the least number that must be subtracted from 4762 that the remainder may be exactly divisible by 93?

24. Multiply 3247 by 999 as shortly as possible.

25. To the excess of 87543 over 9721 add the product of 325 and 173.

26. A torpedo boat leaves Plymouth and runs to Chatham, a distance of 325 miles, at an average speed of 25 miles an hour. How many hours does it take?

27. A good climber can climb 1000 feet in an hour, and can descend 1500 feet an hour. Find the times he would take to ascend and descend a mountain 9000 feet high.

28. If in the division of a number by 315, the operation is performed by short division, and the factors 5, 7, 9 are employed in succession, the remainders being 3, 5, 8 respectively, what is the complete remainder?

PRIME FACTORS

Learn the following:—

A number is divisible by **2** if it is even.

„ „ **3** if the sum of its digits is divisible by 3.

„ „ **4** if the number formed by the last two digits is divisible by 4.

„ „ **5** if it ends with 5 or 0.

„ „ **8** if the number formed by the last three digits is divisible by 8.

„ „ **9** if the sum of its digits is divisible by 9.

Ex. *To find the prime factors of* 4620.

$$\begin{array}{r|l} 2 & 4620 \\ 2 & 2310 \\ 3 & 1155 \\ 5 & 385 \\ 7 & 77 \\ & 11 \end{array}$$

$\therefore\ 4620 = 2^2 \times 3 \times 5 \times 7 \times 11$. **Answer.**

Exercise VIII.

Find the values of :—

1. 3^2+4^2.
2. 4^3+5^2.
3. 5^3+7^2.
4. 6^2-5^2.
5. 7^3-6^2.
6. 4^4+5^4.
7. 8^2-3^3.
8. 9^3+2^4.
9. 8^3+7^3.

Prove that :—

10. $12^2-10^2=2\times22$.
11. $13^2-10^2=3\times23$.
12. $14^2-10^2=4\times24$.
13. $5^3+2^3=7\times19$.
14. $6^3-3^3=3\times63$.
15. $7^3-4^3=3\times93$.
16. $98^2-90^2=8\times188$.
17. $101^2-100^2=201$.
18. $99^2-97^2=2\times196$.
19. $8^2+15^2=17^2$.
20. $2^4+2^2.3^2+3^4=7\times19$.

Find the prime factors of :—

21. 320, 1188.
22. 12600, 16632.
23. 5481, 155034.
24. 1034, 5467.
25. 42900, 18513000.
26. 7875, 21560.
27. 875875, 402325.
28. 346500, 3281850.
29. 1300, 19000.
30. 382239, 28175.
31. 17864, 188100.
32. 37975, 155155.
33. 360, 32400.
34. 16335, 3575.
35. 2464, 379456.
36. 27225, 5967.
37. 34969, 104329.
38. 4356, 2457.
39. 10192, 3808.
40. 67375, 25939375.

HIGHEST COMMON FACTOR

If the given numbers can be easily split into factors, the following method should be adopted :—

Ex. 1. *Find the H.C.F. of* 2926, 20020, 261800.

$$2926 = 2 \times 7 \times 11 \times 19$$
$$20020 = 2^2 \times 5 \times 7 \times 11 \times 13$$
$$261800 = 2^3 \times 5^2 \times 7 \times 11 \times 17$$

$\therefore$ H.C.F. $= 2 \times 7 \times 11 = 154$. Answer.

If the numbers cannot easily be split up into factors, then the *Long Method* should be used. [This method may be deferred, if it is thought advisable.]

Ex. 2. *Find the H.C.F. of* 21535, 31624, 39589.

2	21535	31624	1
	20178	21535	
2	1357	10089	7
	1180	9499	
3	177	590	3
	177	531	
		59	

$\therefore$ The H.C.F. of the first two numbers is 59, and we have now to find the H.C.F. of 59 and 39589.

59	39589	671
	354	
	418	
	413	
	59	
	59	

$\therefore$ H.C.F. $= 59$. Answer.

Exercise IX. (*Oral.*)

Find the H.C.F. of :—

1.	(i) 14, 12.	(ii) 9, 6.	(iii) 7, 21.	(iv) 18, 24.
2.	(i) 6, 18.	(ii) 16, 24.	(iii) 10, 25.	(iv) 22, 55.
3.	(i) 9, 27.	(ii) 20, 30.	(iii) 13, 26.	(iv) 14, 42.
4.	(i) 6, 12, 36.	(ii) 8, 16, 24.	(iii) 4, 20, 36.	(iv) 5, 20, 35.
5.	(i) 5, 25, 45.	(ii) 16, 24, 32.	(iii) 15, 42, 63.	(iv) 28, 32, 20.
6.	(i) 36, 45, 90.	(ii) 35, 56, 84.	(iii) 22, 132, 143.	(iv) 108, 72, 63.

Exercise X.

Find by Factors the H.C.F. of :—

1. 2520, 3024.
2. 84, 90, 100.
3. 234, 306, 414.
4. 945, 10000.
5. 88725, 98735.
6. 495, 1050.
7. 42600, 1760.
8. 100100, 239316.
9. 13, 52, 416, 78.
10. 28, 84, 154, 343.

Find by the 'Long Method' the H.C.F. of :—

11. 1326, 16422.
12. 8987, 14839.
13. 2815, 3941.
14. 10395, 16819.
15. 99789, 56129.
16. 13653, 7585, 50840.
17. 18463, 11359, 7807.
18. 31411, 61889, 124711.
19. 79799, 6817, 16441.
20. 8621, 15611, 24931.

LOWEST COMMON MULTIPLE

Ex. *Find the L.C.M. of* 42, 64, 70, 80, 112.

$$42 = 2 \times 3 \times 7.$$
$$64 = 2^6.$$
$$70 = 2 \times 5 \times 7.$$
$$80 = 2^4 \times 5.$$
$$112 = 2^4 \times 7.$$

$\therefore$ L.C.M. $= 2^6 \times 3 \times 5 \times 7 = 6720$. Answer.

[*Any number which is a factor of one of the other numbers may be neglected in the calculation.*]

If the factors cannot easily be found, then the H.C.F. of the numbers must be obtained by the 'Long Method,' and the numbers thus put into factors.

Exercise XI. (*Oral.*)

Find the L.C.M. of :—

1.	(i) 4, 3, 12.	(ii) 5, 20, 15.	(iii) 3, 5, 4.	(iv) 7, 4, 3.
2.	(i) 5, 10, 15.	(ii) 4, 3, 6.	(iii) 3, 7, 21.	(iv) 8, 9, 36.
3.	(i) 8, 10, 40.	(ii) 8, 12, 15.	(iii) 5, 7, 70.	(iv) 4, 10, 15.
4.	(i) 12, 8, 10.	(ii) 9, 10, 15.	(iii) 5, 6, 7.	(iv) 5, 10, 13.
5.	(i) 7, 9, 63.	(ii) 5, 12, 4.	(iii) 3, 7, 10.	(iv) 3, 4, 10.

Exercise XII.

Find the L.C.M. of :—

1. 25, 35, 42, 60.
2. 16, 18, 20, 21, 28.
3. 15, 24, 25, 27, 30.
4. 36, 56, 63, 84.
5. 6, 8, 15, 16, 20, 24, 25, 36, 40, 96.
6. 18, 16, 36, 44, 45, 48, 63, 121.
7. 39, 20, 6840, 12480.
8. 32, 48, 40, 14.
9. 21, 56, 14, 4.
10. 24, 60, 72, 108.
11. 8, 10, 12, 14, 16, 20, 24, 28.
12. 74, 259.
13. 120, 144, 24, 96.
14. 125, 136, 180, 27, 24.
15. 56, 70, 84, 728.
16. 630, 3528, 12960.
17. 1131, 1885, 2175.
18. 315, 1155, 1287, 2457.
19. 231, 1155, 1617, 2205.
20. 1056, 1980, 2184.
21. 936, 2925.
22. 1512, 700, 18150.
23. 195, 3465, 858.
24. 901, 1219, 13409.
25. 697, 703, 779.
26. 1003, 2301, 4017.
27. 1160, 3875, 3784.

28. If three people whose paces are 20, 32, and 33 inches respectively start walking together, what is the shortest distance, in inches, which will contain the lengths of their paces exactly ?

29. If three bells toll at intervals of 6, 9, and 12 seconds, and start together, after what interval will they be tolling together again ?

30. What is the smallest sum of money that could be paid exactly in shillings, half-crowns, and four-shilling pieces ?

SQUARE AND CUBE ROOT (By Factors)

Ex. 1. *Find the square root of* 1002001 :—

$$\sqrt{1002001} = \sqrt{7^2 \times 11^2 \times 13^2} = 7 \times 11 \times 13 = 1001.$$

Ex. 2. *Find the cube root of* 42875 :—

$$\sqrt[3]{42875} = \sqrt[3]{5^3 \times 7^3} = 5 \times 7 = 35.$$

Exercise XIII. (*Oral.*)

1. What are the square roots of
   36, 144, 49, 64, 25, 100, 196, 225, 256, 169 ?
2. Give the cube roots of 64, 125, 216, 27, 8, 343, 729.
3. Simplify :—
   (i) $\sqrt{25 \times 36}$. (ii) $\sqrt{64 \times 7^2}$. (iii) $\sqrt[3]{8 \times 27}$. (iv) $\sqrt[3]{4^3 \times 3^3}$.

Exercise XIV.

Find by factors the square roots of :—

1. 5184.
2. 28224.
3. 36864.
4. 57600.
5. 69696.
6. 32400.
7. 24336.
8. 213444.
9. 8281.
10. 9801.
11. 5929.
12. 10404.
13. 352836.
14. 51984.
15. 3956121.
16. 585225.
17. 132496.
18. 559504.
19. 736164.
20. 853776.

Find the cube roots of :—

21. 19683 and 970299.
22. 85184 and 17576.
23. 42875 and 166375.
24. 438976 and 12812904.
25. 46656 and 74088.
26. 250047 and 10648.
27. 274625 and 32768.

MONEY, WEIGHTS, AND MEASURES

Reduction

Ex. 1. *Reduce* £3, 5s. 7½d. *to farthings.*

£3, 5s. 7½d.
20
65s.
12
787d.
4
3150 farthings.

Ex. 2. *Reduce* 56203 *pence to pounds.*

12	56203d.	
20	4683s.	7d.
	234£	3s.

∴ 56203d. = £234, 3s. 7d.

Ex. 3. *Reduce* 357601 *grs. to lbs.*

4	357601 grs.		
6	89400	1	1 gr.
20	1490, 0 dwts.	0	
12	745 ozs.		0 dwt.
	62 lbs.		1 oz.

∴ 357601 grs. = 62 lbs. 1 oz. 1 gr.

Ex. 4. *Reduce* 175 qrs. 17 lbs. 8 ozs. *to ounces.*

175 qrs.
28
350
1417
4917 lbs.
16
4917
29510
78680 ozs.

Ex. 5. *Reduce* 76325 lbs. *to cwts.*

4 | 76325 lbs.
7 | 19081 — 1
4 | 2725 qrs. — 6 } 25 lbs.
681 cwts. — 1 qr.

∴ 76325 lbs. = 681 cwts. 1 qr. 25 lbs.

Ex. 6. *Reduce* 62350 inches *to furlongs.*

12 | 62350 ins.
3 | 5195 ft. — 10 ins.
11 | 1731 yds. — 2 ft.
20 | 157 — 4 } 191 yds.
7 fur. — 17

∴ 62350 inches = 7 fur. 191 yds. 2 ft. 10 ins.

Exercise XV. (*Oral.*)

How many pence are there in :—

1. 3s. 6d. ; 2s. 8d. ; 5s. 6d. ; 3s. 2d. ; 5s. 4d. ; 8s. 2d. ?
2. 9s. 6d. ; 10s. 8d. ; 12s. 4d. ; £3, 5s. 0d. ; £2, 2s. 6d. ; £2, 12s. 6d. ?

How many shillings are there in :—

3. £5, 5s. 0d. ; £4, 13s. 0d. ; £6, 10s. 0d. ; £4, 17s. 0d. ; £8, 12s. 0d. ?
4. £10, 10s. 0d. ; £8, 12s. 0d. ; £4, 7s. 0d. ; £6, 10s. 0d. ; £7, 12s. 0d. ?

How many florins are there in :—

5. £5, 10s. 0d. ; £4, 12s. 0d. ; £8, 16s. 0d. ; £9, 18s. 0d. ; £10, 14s. 0d. ?

How many shillings are there, and how many pence over, in :—

6. 145, 37, 52, 45, 98, 72 pence ?
7. 46, 59, 82, 108, 122, 115 pence ?

Exercise XVI.

[*Selections only, from* Exs. xvi. *to* xxviii., *need be worked.*]

Reduce :—

1. £5, 7s. 6d., £3, 2s. 2d., £7, 3s. 9d. to pence.
2. £3, 7s. 2½d., £5, 3s. 9¼d., £3, 14s. 10¾d. to farthings.
3. £8, 7s. 0d., £9, 5s. 0d., £1128, 13s. 0d. to shillings.
4. £3, 8s. 0d., £12, 10s. 0d., £590, 16s. 0d. to florins.
5. £5, 7s. 6d., £9, 12s. 6d., £8, 17s. 6d. to half-crowns.
6. £3, 9s. 6d., £12, 14s. 6d., £8, 7s. 0d. to sixpences.
7. £9, 4s. 3d., £7, 10s. 0d., £11, 14s. 9d. to threepences.
8. £110, 15s. 0d., £93, 10s. 0d., £11, 15s. 0d. to crowns.
9. 12 lbs. 7 ozs. 9 dwts. to dwts.
10. 13 lbs. 9 ozs. 12 dwts. 3 grs. to grains.
11. 9 lbs. 8 ozs. 7 dwts. to dwts.
12. 42 lbs. 7 ozs. 2 dwts. 21 grs. to grains.
13. 51 lbs. 2 ozs. 5 dwts. 18 grs. to grains.
14. 25 lbs. 9 ozs. 5 grs. to grs.
15. 13 lbs. 5 dwts. 18 grs. to grains.
16. 52 lbs. 7 dwts. to dwts.
17. 9 ozs. 22 grs. to grains.
18. 51 lbs. 21 grs. to grains.
19. 50 lbs. 7 dwts. 8 grs. to grains.
20. 72 lbs. 19 dwts. 9 grs. to grains.
21. 3 tons 7 cwts. 3 qrs. to quarters.
22. 1 ton 17 cwts. 2 qrs. 9 lbs. to pounds.
23. 5 tons 3 cwts. 1 qr. 7 lbs. 8 ozs. to ounces.
24. 9 tons 7 lbs. 3 ozs. to drams.
25. 5 bush. 2 pks. 1 gall. 3 qts. to pints.
26. 7 bush. 2 pks. 1 gall. 2 qts. 1 pt. to pints.
27. 3 bush. 2 pks. 3 qts. to pints.
28. 3547 halfpence to pounds.
29. 72315 pence to pounds.
30. 823157 pence to pounds.
31. 8271 florins to pounds.
32. 325 half-crowns to pounds.
33. 8735, 9217, 3572 farthings to pounds.
34. 1721, 7211, 1172 florins to pounds.
35. 152, 187 grs. to dwts
36. 725, 592 grs. to ounces.
37. 15127, 8192 grs. to pounds.
38. 912, 712 dwts. to pounds.
39. 8172, 7182, 8271 grs. to pounds.
40. 540, 721 ozs. to pounds (Avoir.).
41. 727, 843 ozs. to quarters.
42. 2771, 7171 ozs. to cwts.
43. 314591 ozs. to tons.

44. 7241, 4297 drs. to pounds.
45. 8274, 9133 drs. to quarters.
46. 3 cwts. 2 qrs. 9 lbs. to grains.
47. 12 mls. 5 fur. to yards.
48. 5 mls. 4 fur. 7 yds. 2 ft. to feet.
49. 13 mls. 3 fur. 1 ft. to inches.
50. 4 mls. 3 yds. 2 ft. 3 ins. to inches.
51. 57230 ins. to furlongs.
52. 89521 ins. to miles.
53. 102330 ins. to miles.
54. 12 acres 2 rds. to sq. feet.
55. 5 sq. miles to sq. feet.
56. 5 c. yds. to c. inches.
57. 90000 c. ins. to c. yards.
58. 3 sq. mls. 3 rds. to sq. yards.
59. 25 acres 2 rds. 15 per. to perches.
60. 510 acres 3 rds. 21 per. to perches.
61. 540 acres 1 rd. to sq. feet.
62. 82957 sq. in. to sq. yards.
63. 32457 sq. yds. to acres.

EXERCISE XVII.

Miscellaneous Reduction

Reduce :—

1. 17 tons 3 qrs. 17 lbs. to ounces.
2. 158747160 sq. yds. to acres.
3. 189593 grs. to pounds (Troy).
4. 2531 sq. ins. to sq. yards.
5. £8, 12s. 9½d. to halfpence.
6. £4, 13s. 2d. to pence.
7. 3245 pence to pounds.
8. 7211 lbs. to tons.
9. 29005 ft. to miles.
10. £88, 12s. 6d. to half-crowns.
11. £52, 13s. 3d. to threepenny pieces.
12. 59276 florins to pounds.
13. 8 lbs. 9 ozs. 18 dwts. 17 grs. to grains.
14. 11 tons 16 cwts. 3 qrs. 15 lbs. to pounds.
15. 7919 feet to miles.
16. 5 mls. 7 fur. 157 yds. to feet.
17. 832 mls. 5 fur. to poles.
18. 94 pts. to bushels.
19. 61 bush. 2 pks. to quarts.
20. 13 lbs. 8 ozs. 5 dwts. to grains.
21. 3 cwts. 2 qrs. to grains.
22. 5 cwts. 3 qrs. 21 lbs. to grains.
23. £527, 18s. 1½d. to halfpence.
24. 52745 feet to miles.
25. 3 mls. 6 fur. 207 yds. to feet.
26. 3 qrs. 21 lbs. to pounds (Troy).
27. 2 qrs. 15 lbs. to ounces (Troy).
28. 3245 pence to pounds.
29. 3 cwts. 2 qrs. 22 lbs. to grains.
30. £36, 3s. 4½d. to halfpence.
31. 43970 ozs. (Avoir.) to cwts.
32. 52341 ozs. (Avoir.) to cwts.
33. £82, 15s. 9d. to threepences.
34. 359012 guineas to sixpences.
35. 1 ton 11 cwts. 1 qr. 26 lbs. to pounds.
36. 52734 pence to pounds.
37. 427 guineas to pounds.
38. 3245 cwts. to pounds.

39. 26 lbs. (Avoir.) to grains.
40. 42397 farthings to pounds.
41. 7245 pence to pounds.
42. 3 qrs. 21 lbs. to grains.
43. 7249 fourpences to pounds.
44. 8275 half-crowns to pounds.
45. £724, 5s. 9½d. to halfpence.
46. £39, 12s. 6d. to half-crowns.
47. 16 cwts. 2 qrs. 21 lbs. to ounces.
48. 5241 florins to pounds.
49. 9872 half-crowns to pounds.
50. 72415 pence to pounds.
51. 5 tons 4 cwts. 25 lbs. to pounds.
52. £8, 12s. 9¼d. to farthings.
53. £12, 9s. 3d. to threepences.
54. 82596 halfpence to pounds.
55. 4279 crowns to pounds.
56. 17 tons 8 cwts. 13 lbs. to pounds.
57. 18275 pence to pounds.
58. 3 qrs. 17 lbs. to grains.
59. £17, 13s. 9¾d. to farthings.
60. 15 cwts. 2 qrs. 9 lbs. to ounces.
61. £11, 14s. 3d. to threepences.
62. 824971 halfpence to pounds.
63. 42 tons 5 cwts. 7 lbs. to pounds.
64. £117, 14s. 9½d. to halfpence.
65. 7245 crowns to pence.

COMPOUND ADDITION AND SUBTRACTION

Exercise XVIII. (*Oral.*)

Add together :—

1. 5s. 6d., 7s. 2d., 8s. 3d.
2. 5s. 10d., 4s. 7½d., 5s. 5d.
3. 4s. 4d., 5s. 9d., 10s. 10d.
4. £1, 2s. 0d., £3, 5s. 0d., 6s. 8d.
5. £2, 10s. 0d., £1, 12s. 6d., £3, 5s. 0d.
6. 8s. 6d., 15s. 10½d., 10s. 6d.
7. £1, 2s. 6d., 8s. 6d., 10s. 6d.
8. 5s. 8d., 4s. 10½d., £10, 10s. 0d.

Subtract :—

9. 8s. 6d. from 18s. 9d.
10. 4s. 7d. from 13s. 8d.
11. 7s. 6d. from 14s. 9d.
12. 8s. 8d. from 12s. 4d.
13. 15s. 6d. from £1, 0s. 0d.
14. 12s. 2d. from £2, 0s. 0d.
15. £1, 3s. 6d. from £3, 0s. 0d.
16. £1, 4s. 5½d. from £2, 5s. 0d.
17. £2, 11s. 10¼d. from £3, 10s. 0d.
18. £5, 7s. 10d. from £8, 10s. 0d.

Exercise XIX.

Add together :—

1. £3, 11s. 6d., £7, 3s. 2d., £9, 8s. 7d.
2. £3, 19s. 10d., £8, 17s. 2½d., £3, 4s. 9d.
3. £7, 7s. 7d., £8, 8s. 8d., £2, 9s. 3d.
4. £11, 4s. 2½d., £8, 9s. 1½d., £7, 2s. 1½d.
5. £7, 9s. 3½d., £11, 10s. 5½d., £15, 7s. 2½d.
6. £11, 10s. 2¾d., £9, 7s. 1¼d., £8, 2s. 0d.
7. £19, 19s. 11d., £13, 14s. 10¼d., £15, 17s. 10¼d.
8. £210, 7s. 3d., £917, 2s. 4½d., £813, 11s. 11¼d., £912, 8s. 3¾d.
9. £8132, 11s. 2¼d., £7215, 10s. 9½d., £8231, 11s. 10¾d., £702, 15s. 10½d.
10. £7213, 9s. 11¼d., £8213, 10s. 10¾d., £7217, 3s. 11d., £23, 0s. 9¼d.
11. £8219, 19s. 10d., £743, 11s. 0d., £12345, 9s. 8½d., £702, 9s. 3¼d.

12. £7245, 1s. 3¼d., £707, 3s. 2d., £823597, 0s. 1¼d., £23, 19s. 0¼d.
13. £82751, 19s. 11¼d., £5279, 13s. 10¾d., £827, 0s. 2½d., £111, 3s. 9¼d.
14. £80031, 10s. 10½d., £8003, 0s. 3¼d., £700, 1s. 10½d., £807, 3s. 9¼d.
15. £82759, 9s. 9¼d., £2759, 8s. 11d., £927, 8s. 9½d., £297, 9s. 8¼d.
16. 11 lbs. 7 ozs. 9 dwts. 3 grs., 7 lbs. 5 ozs. 19 dwts. 19 grs., 7 lbs. 7 ozs.
17. 12 lbs. 3 ozs. 3 grs., 8 ozs. 7 dwts. 22 grs., 8 ozs. 9 grs.
18. 7 lbs. 7 ozs. 7 dwts. 7 grs., 9 lbs. 2 dwts. 3 grs., 7 lbs. 15 dwts. 15 grs.
19. 21 lbs. 3 ozs. 7 dwts. 9 grs., 5 ozs. 5 dwts. 7 grs., 8 ozs. 10 dwts. 13 grs.
20. 17 lbs. 5 ozs. 6 dwts. 7 grs., 10 lbs. 9 ozs. 18 dwts. 17 grs., 13 lbs. 4 ozs. 7 dwts.
21. 21 lbs. 7 ozs. 9 dwts. 12 grs., 10 lbs. 7 ozs. 9 dwts. 13 grs., 31 lbs. 10 ozs 17 dwts. 21 grs.
22. 17 lbs. 13 ozs., 9 lbs. 12 drs., 16 lbs. 12 ozs. 3 drs., 7 lbs. 13 ozs. 7 drs.
23. 5 cwts. 3 qrs. 7 lbs., 13 cwts. 2 qrs. 21 lbs. 7 ozs., 9 cwts. 17 lbs.
24. 15 cwts. 15 lbs., 7 cwts. 2 qrs. 5 ozs., 19 cwts. 2 qrs. 25 lbs. 15 ozs.
25. 17 cwts. 14 lbs., 6 cwts. 1 qr. 21 lbs., 5 cwts. 3 qrs. 7 lbs., 9 cwts. 12 lbs.
26. 3 tons 15 cwts. 12 lbs. 5 ozs., 7 tons 13 cwts. 14 lbs. 8 ozs., 14 cwts. 1 qr. 4 lbs.
27. 1 cwt. 1 qr. 14 lbs., 5 cwts. 2 qrs. 16 lbs., 10 cwts. 2 qrs. 6 lbs., 5 cwts. 1 qr. 14 lbs.
28. 52 yds. 2 ft. 3 ins., 105 yds. 7 ins., 72 yds. 2 ft.
29. 200 yds. 2 ft. 11 ins., 2 ft. 10 ins., 208 yds. 1 ft. 9 ins.
30. 7 fur. 9 ins., 6 fur. 200 yds. 2 ft. 3 ins., 5 fur. 2 ft. 3 ins.
31. 5 mls. 6 fur., 12 mls. 3 fur. 2 yds., 8 yds. 2 ft. 10 ins.
32. 12 mls. 3 fur. 7 yds., 8 yds. 2 ft. 10 ins., 5 fur. 2 ft. 11 ins.
33. 3 acs. 2 rds. 1000 sq. yds. 2 sq. ft. 100 sq. ins., 3 rds. 150 sq. yds. 4 sq. ft. 10 sq. ins., 900 sq. yds. 70 sq. ins.
34. 5 acs. 3 rds. 960 sq. yds. 7 sq. ft., 21 acs. 105 sq. yds. 3 sq. ft., 4 rds. 990 sq. yds. 7 sq. ft. 120 sq. ins.
35. 51 acs. 100 sq. ins., 3 rds. 3 sq. ft., 70 sq. yds. 8 sq. ft., 8 sq. ft. 100 sq. ins.

Exercise XX.

Subtract :—

1. 1s. 3d. from 2s. 9d., and 2s. 6d. from 7s. 10d.
2. 8s. 2d. from 19s. 11d., and 7s. 3d. from 12s. 9d.
3. 8s. 10½d. from £1, 10s., and 7s. 9¼d. from 10s. 3¼d.
4. £1, 5s. 7½d. from £2, 3s. 4¼d.
5. £3, 11s. 10d. from £4, 13s. 5¼d.
6. £12, 13s. 11¼d. from £13, 9s. 9¼d.
7. £123, 19s. 7¼d. from £135, 2s. 2d.
8. £825, 19s. 11¾d. from £892, 13s. 5d.
9. £7295, 19s. 9d. from £8230, 13s. 7½d.
10. £8235, 17s. 7½d. from £8372, 15s. 2d.
11. £8181, 18s. 11¼d. from £8228, 17s. 2½d.
12. £12345, 7s. 7½d. from £23245, 17s. 11d.
13. £81823, 19s. 1¼d. from £123456, 13s. 4d.
14. £72741, 13s. 9¾d. from £314711, 2s. 2½d.
15. 11 ozs. 7 dwts. from 1 lb. 7 dwts. 2 grs.
16. 3 lbs. 7 ozs. 19 dwts. from 5 lbs. 3 ozs. 7 dwts.
17. 5 lbs. 17 dwts. 8 grs. from 7 lbs. 2 ozs. 5 dwts. 5 grs.
18. 5 lbs. 3 ozs. 14 dwts. 13 grs. from 8 lbs. 7 ozs. 7 dwts. 7 grs.
19. 21 lbs. 7 dwts. 7 grs. from 29 lbs. 11 ozs.
20. 13 lbs. 14 dwts. from 15 lbs. 7 ozs. 7 dwts.
21. 29 lbs. 9 ozs. 9 grs. from 32 lbs. 3 ozs. 3 dwts.
22. 3 qrs. 7 lbs. from 1 cwt. 8 lbs.
23. 5 cwts. 2 qrs. 21 lbs. 3 ozs. from 1 ton 2 qrs. 20 lbs. 7 ozs.
24. 4 cwts. 3 qrs. 20 lbs. 11 ozs. from 8 cwts. 2 qrs. 22 lbs. 15 ozs.
25. 5 cwts. 7 lbs. 7 ozs. from 1 ton 2 cwts. 2 qrs. 13 lbs.
26. 7 cwts. 9 lbs. 3 ozs. from 18 cwts. 17 lbs. 15 ozs.
27. 21 tons 3 cwts. 3 qrs. 5 lbs. from 29 tons 5 cwts. 7 lbs.
28. 9 pks. 3 qts. 1 pt. from 21 pks. 3 qts.

29. 32 pks. 1 gall. 3 qts. from 43 pks. 1 gall. 1 qt.
30. 100 yds. 2 ft. 7 ins. from 150 yds. 1 ft. 8 ins.
31. 52 yds. 2 ft. 9 ins. from 60 yds. 1 ft. 1 in.
32. 3 fur. 100 yds. 3 ins. from 5 fur. 70 yds. 2 ft. 10 ins.
33. 4 mls. 2 fur. 115 yds. 2 ft. 11 ins. from 8 mls. 2 yds. 3 ins.
34. 72 sq. yds. 5 sq. ft. 7 sq. ins. from 80 sq. yds. 4 sq. ft. 100 sq. ins.
35. 51 sq. yds. 2 sq. ft. 100 sq. ins. from 72 sq. yds. 5 sq. ins.
36. 890 sq. yds. 8 sq. ft. 3 sq. ins. from 1 ac. 2 rds.
37. 2 rds. 900 sq. yds. 108 sq. ins. from 2 acs. 5 sq. ft.

Miscellaneous Compound Addition and Subtraction

Exercise XXI.

Add together :—

1. £8, 17s. 9½d., £4, 12s. 3d., £5, 9s. 9d., £7, 2s. 2¼d.
2. £14, 17s. 10½d., 9s. 3d., £4, 11s. 2d., £8, 0s. 3¾d.
3. £9, 12s. 10¾d., £8, 7s. 5d., £9, 10s. 10½d., 7s. 7d.
4. 9 lbs. 10 ozs. 19 dwts. 20 grs., 8 lbs. 5 ozs. 18 dwts. 15 grs., 2 lbs. 7 ozs. 10 dwts. 5 grs.
5. 51 tons 4 cwts. 3 qrs. 20 lbs. 4 ozs., 4 tons 2 qrs. 17 lbs., 50 tons 1 qr. 5 lbs.
6. £8, 12s. 9¼d., £91, 7s. 10½d., £42, 0s. 0¼d., £7, 7s. 7d.
7. £42, 13s. 9½d., £17, 2s. 0¾d., £4, 4s. 11½d., 9s. 11¼d.
8. 53 lbs. 4 ozs. 5 dwts. 6 grs., 19 lbs. 10 ozs. 10 dwts., 5 ozs. 15 dwts. 8 grs., 31 lbs. 4 ozs. 4 dwts. 4 grs.
9. 92 tons 15 cwts. 3 qrs. 5 ozs., 87 tons 14 cwts. 2 qrs. 20 lbs. 13 ozs., 901 tons 10 cwts. 15 lbs. 12 ozs., 42 tons 12 cwts. 1 qr. 14 lbs. 5 ozs.
10. £8, 12s. 9½d., £19, 7s. 4¼d., £15, 15s. 10½d., 7s. 9¼d.
11. £823, 7s. 10½d., £924, 9s. 9d., £1024, 7s. 2¼d., £802, 0s. 3d.
12. £1023, 19s. 10½d., £405, 17s. 4d., £1592, 9s. 11¼d., £724, 13s. 4½d., £8275, 0s. 7¼d.

13. 102 lbs. 11 ozs. 17 dwts. 15 grs., 31 lbs. 5 ozs. 15 dwts. 14 grs., 20 lbs. 10 ozs. 10 dwts. 10 grs., 70 lbs. 3 ozs. 1 dwt. 18 grs.

14. 42 tons 19 cwts. 2 qrs. 5 ozs., 53 tons 4 cwts. 3 qrs. 10 lbs. 4 ozs., 81 tons 9 cwts. 2 qrs. 10 lbs.

15. £9047, 15s. 9½d., £8251, 4s. 11d., £7241, 15s. 4¼d., £10072, 3s. 0¼d.

16. £42579, 15s. 4½d., £8241, 3s. 10d., £52700, 19s. 8¼d., £8250, 14s. 7d., £20009, 10s. 3¼d.

17. 14 tons 14 cwts. 2 qrs. 14 lbs., 361 tons 15 cwts. 3 qrs. 5 lbs., 94 tons 9 cwts. 1 qr. 19 lbs., 325 tons 6 cwts. 2 qrs. 20 lbs.

18. 9 lbs. 2 ozs. 12 dwts. 10 grs., 7 lbs. 5 ozs. 14 dwts. 6 grs., 12 lbs. 10 ozs. 9 dwts. 8 grs., 3 lbs. 1 oz. 15 dwts. 20 grs., 6 lbs. 7 ozs. 17 dwts. 18 grs.

19. £1047, 16s. 4¾d., £1, 0s. 9½d., 14s. 8¼d., £62, 0s. 1¼d.

20. £96, 10s. 3¼d., £78, 19s. 11¾d., £56, 7s. 8½d., £35, 18s. 7½d., £109, 8s. 3¼d., £268, 12s. 10¾d.

Subtract :—

21. £52, 7s. 9½d. from £85, 11s. 5d.

22. £82, 5s. 7¼d. from £90.

23. 4 tons 5 cwts. 20 lbs. from 5 tons.

24. 12 lbs. 8 ozs. 4 dwts. 2 grs. from 13 lbs. 7 ozs.

25. £425, 17s. 11¼d. from £824, 17s.

26. 5 tons 17 cwts. 2 qrs. 15 lbs. from 7 tons 14 cwts. 2 qrs. 10 lbs.

27. £391, 15s. 6¾d. from £729, 12s. 8½d.

28. 114 tons 11 cwts. 5 lbs. from 115 tons 2 cwts. 2 lbs.

29. 1 ton 7 cwts. 3 qrs. 17 lbs. from 10 tons 5 cwts. 2 qrs. 14 lbs.

30. £456, 13s. 5¾d. from £562, 10s. 3½d.

31. 7 cwts. 3 qrs. 26 lbs. from 15 cwts. 3 qrs. 18 lbs.

32. £7892, 14s. 7½d. from £1047983, 17s. 6¾d.

33. 5 tons 2 qrs. 7 lbs. from 6 tons 9 cwts. 1 qr. 5 lbs.

COMPOUND MULTIPLICATION

Ex. 1. *Multiply* £3, 4s. 5¼d. *by* 77.

£	s.	d.	
3	4	5¼	
		7	
22	11	0¾	
		11	
£248	1	8¼	Answer.

[*Rough Check:* £3, 4s. 0d. × 80 = £256.]

If the multiplier cannot be split up into a small number of simple factors, then one of the following methods may be used :—

Ex. 2. *Multiply* 5 cwts. 2 qrs. 17 lbs. 5 ozs. *by* 379.

METHOD I.

Tons	cwts.	qrs.	lbs.	ozs.	
	5	2	17	5	
				10	
2	16	2	5	2	= 10 times
				10	
28	5	1	23	4	=100 ,,
				3	
84	16	1	13	12	=300 ,,
19	15	3	7	14	= 70 ,,
2	10	3	15	13	= 9 ,,
107	3	0	9	7	=379 times

METHOD II.

(i) 379 × 5

4 | 1895 ozs.
4 | 473 — 3
118 lbs. — 1 } 7 ozs.

(ii) 379 × 17

379
2771 (add 118)

4 | 6561 lbs.
7 | 1640 — 1
234 qrs. — 2 } 9 lbs.

(iii) 379 × 2 (add 234)

4 | 992 qrs.
248 cwts. 0 qrs.

(iv) 379 × 5 (add 248)

20 | 2143 cwts.
107 tons 3 cwts.

Product = 107 tons 3 cwts. 9 lbs. 7 ozs.

In (i) multiply the number of ozs. by 379 and reduce to lbs.

(ii) multiply the number of lbs. by 379, add the number of lbs. carried from (i) and reduce to qrs.

(iii) multiply the number of qrs. by 379, add the number of qrs. from (ii) and reduce to cwts.

(iv) multiply the number of cwts. by 379, add the number of cwts. from (iii) and reduce to tons.

Exercise XXII. (*Oral.*)

Multiply :—

1. 5s. 6d. by 4.
2. 13s. 4d. by 3.
3. 12s. 8d. by 9.
4. 7s. 2d. by 8.
5. 8s. 4d. by 5.
6. 2s. 7d. by 11.
7. 5s. 5d. by 12.
8. 7s. 2d. by 7.
9. 10s. 2d. by 8.
10. £2, 10s. 6d. by 6.
11. £5, 10s. 0d. by 5.
12. £2, 5s. 6d. by 7.
13. 4 lbs. 3 ozs. 7 dwts. by 5.
14. 2 qrs. 5 lbs. by 8.
15. 5 yds. 2 ft. by 11.

Exercise XXIII.

Multiply :—.

1. £2, 15s. 11d. by 55 and 32.
2. £7, 10s. 3½d. by 24 and 36.
3. £6, 12s. 9¼d. by 42 and 44.
4. £8, 3s. 10½d. by 48 and 60.
5. £13, 10s. 9d. by 72 and 45.
6. £121, 17s. 4½d. by 125 and 343.
7. £142, 10s. 3¼d. by 216 and 512.
8. £215, 7s. 3d. by 252 and 150.
9. £221, 10s. 9d. by 105 and 140.
10. £237, 4s. 6d. by 315 and 280.
11. £202, 7s. 8d. by 141 and 163.
12. £137, 11s. 9d. by 101 and 113.
13. £325, 19s. 7d. by 83 and 127.
14. £14, 3s. 8¼d. by 347.
15. £75, 14s. 0¾d. by 127.
16. £7, 16s. 11½d. by 407.
17. £734, 14s. 10¼d. by 347.
18. 11 ozs. 19 dwts. 7 grs. by 42 and 81.
19. 15 lbs. 5 ozs. by 54 and 77.
20. 9 lbs. 7 ozs. 5 dwts. by 56 and 35.
21. 10 lbs. 9 ozs. 15 dwts. 17 grs. by 112 and 110.
22. 21 lbs. 5 ozs. 10 dwts. 12 grs. by 56 and 121.
23. 17 lbs. 11 ozs. 9 dwts. 13 grs. by 72 and 96.
24. 6 ozs. 11 dwts. 10 grs. by 431.
25. 12 lbs. 3 ozs. 7 dwts. 15 grs. by 127.
26. 13 lbs. 10 ozs. 16 dwts. 19 grs. by 223.
27. 15 lbs. 11 ozs. 17 dwts. 9 grs. by 233.
28. 5 cwts. 2 qrs. 4 lbs. by 35 and 72.
29. 1 ton 2 cwts. 3 qrs. 15 lbs. by 40 and 63.
30. 3 tons 7 cwts. 2 qrs. 12 lbs. 8 ozs. by 60 and 72.
31. 5 tons 14 cwts. 1 qr. 7 ozs. by 30 and 432.
32. 9 tons 15 cwts. 15 lbs. 8 ozs. by 81 and 225.
33. 5 cwts. 3 qrs. 7 lbs. by 73, 89 and 241.
34. 81 tons 9 cwts. 2 qrs. 21 lbs. by 79, 203 and 113.
35. 32 tons 3 cwts. 1 qr. 20 lbs. 8 ozs. by 47, 89 and 119.
36. 50 yds. 2 ft. 7 ins. by 30, 35 and 42.
37. 3 fur. 100 yds. 3 ins. by 55, 63 and 72.
38. 1 ml. 2 fur. 5 yds. 2 ft. 7 ins. by 79, 89 and 97.
39. 50 sq. yds. 7 sq. ft. 100 sq. ins. by 121, 84 and 96.
40. 2 acs. 3 rds. 250 sq. yds. by 259 and 331.

Miscellaneous Compound Multiplication

EXERCISE XXIV.

Multiply :—

1. £8, 3s. 10½d. by 35 and 63.
2. 7 cwts. 2 qrs. 23 lbs. by 72 and 99.
3. 12 lbs. 5 ozs. 15 dwts. by 59 and 73.
4. 59 yds. 2 ft. 9 ins. by 77 and 132.
5. £7, 12s. 11¾d. by 31 and 119.
6. 12 acs. 2 rds. 570 sq. yds. by 42 and 123.
7. 32 lbs. 12 dwts. 4 grs. by 99 and 108.
8. 5 tons 14 cwts. 2 qrs. 15 lbs. by 37 and 127.
9. £115, 11s. 10½d. by 233 and 331.
10. 5 mls. 7 fur. 32 po. by 144 and 159.
11. 3 tons 2 qrs. 21 lbs. by 121 and 137.
12. 5 lbs. 10 ozs. 12 dwts. by 53 and 235.
13. 520 acs. 2 rds. 30 sq. po. by 66 and 217.
14. £7203, 11s. 2¾d. by 396 and 343.
15. 8 tons 7 cwts. 23 lbs. by 512 and 729.
16. 1 qr. 5 lbs. 8 ozs. by 139.
17. 18 cwts. 15 lbs. by 503.
18. 70 yds. 2 ft. 10 ins. by 29.
19. 5 sq. yds. 8 sq. ft. 131 sq. ins. by 80.
20. £8, 7s. 6½d. by 107.
22. 116 sq. yds. 7 sq. ft. 41 sq. ins. by 58.
23. 7 lbs. 10 ozs. 18 dwts. 21 grs. by 92.
24. £3, 2s. 7d. by 135.
25. £5, 17s. 8¾d. by 1163.
26. £3, 17s. 10d. by 7234.
27. 3 acs. 2 rds. 21 sq po. by 297
28. 5 qrs. 7 bush. 2 pks. by 865.
29. £20, 18s. 4½d. by 308.
30. 9s. 3¾d. by 501.
31. £1, 15s. 8¼d. by 150.
32. 5 mls. 6 fur. 115 yds. by 44.
33. 17s. 9½d. by 1296.
34. 7 cwts. 1 qr. 9 lbs. 6 ozs. by 132.
35. £9, 0s. 11½d. by 144.
36. 2 ozs. 1 dwt. 19 grs. by 71.
37. £5, 14s. 4¼d. by 304.
38. £1, 9s. 6d. by 339.
39. £94, 19s. 3¾d. by 251.
40. £451, 15s. 8¼d. by 239.
41. 57 lbs. 5 ozs. 15 dwts. 10 grs. by 629.
42. £235, 10s. 5¼d. by 263.
43. £305, 0s. 7½d. by 281.
44. 102 lbs. 8 ozs. 19 dwts. 20 grs. by 731.
45. £502, 9s. 11¾d. by 353.
46. £7027, 15s. 4½d. by 629.
47. 150 lbs. 15 dwts. 10 grs. by 773.
48. £450, 10s. 9¾d. by 293.
49. 215 tons 15 cwts. 2 qrs. 20 lbs. by 851.

COMPOUND DIVISION

Ex. 1. *Divide* £1722, 14s. 6d. *by* 24.

$$\left\{\begin{array}{c|l} 4 & £1722,\ 14s.\ 6d. \\ \hline 6 & 430,\ 13s.\ 7\tfrac{1}{2}d. \\ \hline & £71,\ 15s.\ 7\tfrac{1}{4}d. \end{array}\right.$$

[*Rough Check*: £1725 ÷ 25 = £69.]

If the divisor cannot be split up into a small number of simple factors, then the following method may be used :—

Ex. 2. *Divide* £1422, 3s. 10½d. *by* 727.

£1, 19s. 1½d. Quotient

727)£1422, 3s. 10½d.
727
695
20
13903 s.
727
6633
6543
90
12
1090 d.
727
363
2
727 half-pence
727

[*Rough Check*: £1420 ÷ 730 = £2 (approx.).]

Exercise XXV. (*Oral.*)

Divide :—

1. £1, 1s. 0d. by 4.
2. 7s. 6d. by 3.
3. 13s. 9d. by 5.
4. 15s. 6d. by 6.
5. £1, 3s. 4d. by 7.
6. £1, 2s. 0d. by 8.
7. £1, 11s. 8d. by 10.
8. 12s. by 9.
9. 15s. 6¾d. by 3.
10. 16s. 5d. by 4.
11. 17s. 8½d. by 10.
12. 11s. 5½d. by 5.

Exercise XXVI.

Divide :—

1. £57, 5s. 10d. by 10.
2. £358, 1s. 7d. by 14.
3. £779, 17s. 0d. by 72.
4. £4571, 2s. 0d. by 96.
5. £5975, 13s. 4d. by 64.
6. £2170, 6s. $2\frac{3}{4}$d. by 121.
7. £43084, 6s. 10d. by 88.
8. £174, 14s. $0\frac{3}{4}$d. by 45.
9. £3983, 14s. 0d. by 144.
10. £3512, 5s. 0d. by 360.
11. £222, 13s. $7\frac{1}{2}$d. by 79.
12. £2683, 2s. 8d. by 241.
13. £1014, 10s. $10\frac{3}{4}$d. by 127.
14. £3007, 15s. $8\frac{1}{2}$d. by 443.
15. £68866, 4s. $4\frac{3}{4}$d. by 193.
16. £13144, 7s. $10\frac{1}{4}$d. by 151.
17. 9 lbs. 11 ozs. 9 dwts. 12 grs. by 18.
18. 1135 lbs. 12 dwts. by 84.
19. 353 lbs. 7 ozs. 15 dwts. by 97.
20. 1591 lbs. 8 ozs. 16 dwts. by 127.
21. 12 lbs. 2 ozs. 13 dwts. 22 grs. by 17.
22. 500 lbs. 2 ozs. 14 dwts. 16 grs. by 532.
23. 107 tons 10 cwts. 12 lbs. by 11.
24. 21 tons 15 cwts. 3 qrs. 21 lbs. by 25.
25. 38 tons 1 cwt. 3 qrs. by 44.
26. 1 ton 7 cwts. 1 qr. 24 lbs. 11 ozs. 8 drs. by 140.
27. 100 tons 9 cwts. by 112.
28. 57 tons 17 cwts. 2 qrs. 5 lbs. by 129.
29. 1 fur. 7 yds. 2 ft. 11 ins. by 29.
30. 26 mls. 4 fur. 184 yds. by 36.
31. 73 mls. 7 fur. 200 yds. 1 ft. by 241.
32. 20190 acs. 1 rd. 16 sq. po. by 53.
33. 699 acs. 35 sq. po. by 125.
34. 1718 acs. 2 rds. 620 sq. yds. by 216.

Miscellaneous Compound Division

EXERCISE XXVII.

Divide :—

1. £284067, 11s. 8d. by 56.
2. £34118, 4s. $10\frac{3}{4}$d. by 31.
3. 14841 galls. 3 qts. by 257.
4. 53 lbs. 3 ozs. 7 dwts. 12 grs. by 150.
5. 6356 acs. 1 rd. by 600.
6. 69 tons 17 cwts. 2 qrs. 7 lbs. by 413.
7. 121 lbs. 3 ozs. 16 dwts. 2 grs. by 23.
8. 247 mls. 1 fur. 210 yds. by 45.
9. £69218, 2s. 0d. by 48.
10. 4729 acs. 659 sq. yds. 7 sq. ft. 18 sq. ins. by 54.
11. 29 tons, 6 cwts. 3 qrs. 11 lbs. by 218.
12. 345 mls. 7 furs. 111 yds. 1 ft. by 317.
13. 9 lbs. 11 ozs. 9 dwts. 12 grs. by 54.
14. 177 tons 7 cwts. 27 lbs. 4 ozs. 9 drs. by 197.
15. £1055, 19s. $9\frac{3}{4}$d. by 657.
16. 13960 bush. 2 pks. by 227.
17. 491 mls. 5 fur. 90 yds. 2 ft. 2 ins. by 23.
18. 1475 lbs. 8 ozs. 10 dwts. by 107.
19. 5524 tons 7 cwts. 2 qrs. 25 lbs. by 75.
20. £3349, 1s. 5d. by 34.
21. 1727 cu. yds. 26 cu. ft. by 1728.
22. 211 mls. 1 fur. 75 yds. by 27.
23. £1781, 5s. 0d. by 125.
24. 295 tons 19 cwts. 1 qr. 18 lbs. 8 ozs. by 408.
25. 139 yds. 2 ft. 4 ins. by 37.
26. £34806, 0s. $2\frac{3}{4}$d. by 841.
27. £25932, 13s. $3\frac{1}{2}$d. by 19.
28. 271 yds. 1 ft. 7 ins. by 17.
29. £86051, 8s. 9d. by 63.
30. 19055 lbs. 4 ozs. 11 dwts. 12 grs. by 353.
31. £13152, 13s. 4d. by 181.
32. £7529, 16s. 6d. by 173.
33. 38579 lbs. 1 oz. 12 grs. by 847.
34. £821, 2s. 5d. by 86.
35. 359 lbs. 4 ozs. (Troy) by 96.
36. 69 tons 17 cwts. 2 qrs. 7 lbs. by 826.
37. £18799, 8s. $11\frac{1}{2}$d. by 3715.
38. 228 tons 18 cwts. 3 qrs. 13 lbs. 12 ozs. by 76.
39. £4694, 0s. $0\frac{3}{4}$d. by 59.
40. £509, 2s. $4\frac{1}{2}$d. by 42.
41. 8 tons 18 cwts. 3 qrs. 14 lbs. by 53.

42. £120043, 18s. 8d. by 64.
43. £12, 18s. $4\frac{1}{2}$d. by 39.
44. 32 tons 1 cwt. 2 qrs. 2 lbs. by 75.
45. 30 tons 6 cwts. 1 qr. by 35.
46. £165, 15s. $8\frac{3}{4}$d. by 139.
47. £646, 16s. $4\frac{1}{2}$d. by 54.
48. £6154, 1s. $7\frac{3}{4}$d. by 977.
49. 93 tons 19 cwts. 3 qrs. 14 lbs. by 557.
50. £3683, 13s. $6\frac{1}{2}$d. by 1031.

Compound Division—*continued*

Ex. 1. *How many times is* £56, 16s. $9\frac{3}{4}$d. *contained in* £4945, 2s. $8\frac{1}{4}$d. ?

£4945, 2s. $8\frac{1}{4}$d.	£56, 16s. $9\frac{3}{4}$d.	
20	20	
98902 s.	1136 s.	87
12	12	54567)4747329
1186832 d.	13641 d.	436536
4	4	381969
4747329 farthings	54567 farthings	381969

$\therefore$ Quotient $= \frac{4747329 \text{ farthings}}{54567 \text{ farthings}} = 87.$

[*Rough Check* : $\frac{£4945}{£55} = 90$ (approx.).]

Exercise XXVIII.

How many times is the second of the following quantities contained in the first :—

1. £17, 11s. 9d., £2, 18s. $7\frac{1}{2}$d.
2. £45, 12s. 9d., £3, 16s. $0\frac{3}{4}$d.
3. £1043, 7s. $5\frac{1}{4}$d., £347, 15s. $9\frac{3}{4}$d.
4. £58402, 15s. $5\frac{1}{4}$d., £721, 0s $5\frac{1}{4}$d.
5. £9961, 7s. $6\frac{1}{2}$d., £35, 16s. $7\frac{3}{4}$d.
6. £4152, 1s. $10\frac{1}{2}$d., £65, 18s. $1\frac{1}{2}$d.
7. £587, 4s. 3d., £83, 17s. 9d.
8. £1299, 19s. $3\frac{3}{4}$d., £4, 13s. $2\frac{1}{4}$d.
9. £1213, 6s. 8d., £10, 16s. 8d.
10. £4536, 17s. $7\frac{1}{4}$d., £5, 10s. $6\frac{1}{4}$d.
11. £5688, 15s. 3d., £20, 12s. $2\frac{3}{4}$d.
12. £3651, 11s. 3d., £1, 3s. $5\frac{1}{4}$d.
13. £500, 0s. 6d., £1, 9s. 6d.

14. £2055, 18s., £1, 13s. 4½d.
15. £1199, 13s. 4½d., £3, 7s. 2½d.
16. 26 lbs. 7 ozs. 10 dwts., 5 lbs. 3 ozs. 18 dwts.
17. 1575 lbs. 3 ozs. 3 dwts., 13 lbs. 2 ozs. 17 dwts.
18. 944 lbs. 10 ozs. 15 dwts., 4 lbs. 10 ozs. 15 dwts.
19. 386 lbs. 11 ozs. 3 dwts. 11 grs., 16 lbs. 9 ozs. 17 dwts. 13 grs.
20. 362 lbs. 5 ozs. 14 dwts. 9 grs., 8 lbs. 13 dwts. 5 grs.
21. 346 lbs. 4 ozs. 7 dwts. 23 grs., 8 lbs. 13 dwts. 5 grs.
22. 16 lbs. 3 ozs. 4 dwts., 5 dwts. 8 grs.
23. 114 tons 10 cwts. 1 qr. 13 lbs., 3 tons 1 cwt. 3 qrs. 17 lbs.
24. 489 tons 14 cwts. 1 qr., 5 tons 16 cwts. 2 qrs. 11 lbs.
25. 580 tons 10 cwts. 2 qrs., 3 tons 15 cwts. 1 qr. 16 lbs.
26. 933 tons 9 cwts. 2 qrs. 1 lb., 4 tons 15 cwts. 2 qrs. 27 lbs.
27. 990 tons 7 cwts. 2 qrs., 17 tons 13 cwts. 2 qrs. 23 lbs.
28. 135 tons 4 cwts. 3 qrs. 14 lbs., 3 tons 4 cwts. 1 qr. 17 lbs.
29. 53 tons 3 cwts. 1 qr. 15 lbs. 6 ozs., 7 cwts. 1 qr. 9 lbs. 6 ozs.
30. 11 tons 11 cwts. 3 ozs., 15 cwts. 1 qr. 16 lbs. 13 ozs.
31. 2 cwts. 3 qrs. 24 lbs. 8 ozs., 17 lbs. 8 ozs.
32. 561 tons 19 cwts. 1 qr. 4 lbs., 4 tons 13 cwts. 2 qrs. 18 lbs.
33. 28 tons 19 cwts. 2 qrs. 13 lbs., 3 tons 4 cwts. 1 qr. 17 lbs.
34. 7 tons 11 cwts. 2 qrs. 19 lbs., 18 cwts. 3 qrs. 23 lbs. 6 ozs.
35. 162 tons 10 cwts., 1 cwt. 13 lbs.
36. 1 ton 7 cwts. 3 qrs., 3 qrs.

Miscellaneous Problems on Compound Rules

Exercise XXIX.

1. Subtract the sum of £53, 2s. 11d., £72, 8s. 9d., and £42, 17s. 3d. from £200.
2. From three times £42, 17s. 6d. take five times £17, 4s. 11½d.
3. A man possessing £400 pays bills of £37, 4s. 9d., £42, 15s. 7d., £142, 18s. 6d. How much has he left?
4. What sum must be added to £37, 19s. 11¼d. to make 10,000 pence?
5. Divide £25, 8s. 7½d. between 2 men, so that one may have twice as much as the other.

6. Out of a sum of money, 17 men received 3s. $4\frac{1}{2}$d. each, and there remained 2s. $5\frac{3}{4}$d. How much was the original sum ?

7. Divide £15942, 16s. 6d. by 108. If you had to distribute this sum of money equally among 216 persons, what sum would each receive ?

8. A farmer sells 259 sheep at £2, 2s. each, and buys 14 bullocks at £27, 10s. each, and 13 at £22, 5s. each. How much does he gain or lose ?

9. If I spend £1, 18s. 6d. per week for rent and taxes, £5, 3s. for housekeeping, £1, 15s. for clothes, and £2, 3s. 6d. for sundries, for how many weeks will £407 last me ?

10. An equal number of men, women, and boys earn £129, 18s. 9d. in five weeks ; if each man earns 3s. 9d., each woman 2s. 9d., and each boy 1s. 9d. per day, how many were there of each ?

11. A postman, whose pay for a week is 15s., is fined 1s. 6d. if he comes in late, and at the end of 13 weeks he receives £8, 15s. 6d. How often was he late ?

12. In 42 days a man earns £6, 0s. 9d. How much does he earn per day ?

13. How often is the sum of a farthing, sixpence, a florin, and a half-crown contained in £26, 2s. 2d. ?

14. How many yards of silk at 8s. $0\frac{1}{4}$d. a yard must be given in exchange for 230 pounds of tea at 3s. $2\frac{1}{2}$d. a pound ?

15. Two persons have £81, 17s. 4d. between them, and one has £3, 7s. 6d. more than the other. How much does each have ?

16. The total weight of Bow Bells (10 in number) is 10 tons 5 cwts. ; find the average weight of each bell.

17. Find the cost of covering the floor of a room containing 54 sq. yds. 3 sq. ft. at 8d. per sq. ft.

18. The circumferences of the wheels of a bicycle are 88 and 94 inches respectively ; how many times will each turn during a distance of 114 yds. 2 ft. 8 ins. ?

9. How long will 54 tons of coal last for 5 fires, if each consumes 18 lbs. per day?

0. How many pieces of cloth, each 5 yds. 2 ft. long, can be cut from a roll of length 80 yds., and how much is left over?

1. Find the number of days between March 1st, 1884, and August 1st, 1886, including one of the days mentioned.

2. Divide 300 tons of coal equally among 84 people.

3. Divide 14 tons 17 cwts. 3 qrs. 9 lbs. 8 ozs. 4 drs. of goods among 63 people. Show that the value of each person's share at a penny an ounce will be £35, 5s. $11\frac{3}{4}$d.

4. Along a certain path 598 yds. 1 ft. long, stakes are placed at intervals of 5 feet. How many stakes will be needed, including the 2 extreme stakes?

5. If 1 cwt. 3 qrs. of tea is made up into parcels containing 1 lb. 8 ozs., 2 lbs. 8 ozs., and 3 lbs. respectively, an equal number of each, how many parcels will there be altogether?

DECIMALS AND THE METRIC SYSTEM

10 millimetres = 1 centimetre,

and 1 millimetre is called *one-tenth* (written $\frac{1}{10}$) of 1 centimetre ;

100 millimetres = 1 decimetre,

and 1 millimetre is called *one-hundredth* (written $\frac{1}{100}$) of 1 decimetre.

Similarly 1 millimetre is *one-thousandth* (written $\frac{1}{1000}$) of 1 metre.

In just the same way that

523 means 5 *hundreds*, 2 *tens* and 3 *units*,

so, in the decimal notation,

5·23 means 5 *units*, 2 *tenths*, 3 *hundredths*,

504·107 means 5 *hundreds*, 4 *units*, 1 *tenth*, 7 *thousandths*,

the digits to the left of the decimal point representing multiples of the unit, and those to the right of the decimal point submultiples.

Thus the methods of performing Addition, Subtraction, etc., of numbers involving a decimal point will be identical with the processes already studied, the decimal point merely indicating the position of the unit.

Ex. 1. 82·57 metres = (8 tens 2 units 5 tenths 7 hundredths) of metres.
= 8 Dm. 2 m. 5 dm. 7 cm.

Ex. 2. 7 Hg. 5 g. 7 cg. 8 mg. = (7 hundreds 5 units 7 hundredths 8 thousandths) of grams.
= 705·078 gm.

Exercise XXX. (*Oral.*)

Express at full length :—

1. 324·21 metres.	**6.** 127·403 metres.	**11.** 10·002 grams.
2. 14·05 litres.	**7.** 200·701 litres.	**12.** 127·05 metres.
3. 104·072 grams.	**8.** 109·008 grams.	**13.** 2478·5 litres.
4. 123·413 metres.	**9.** 18·213 metres.	**14.** 18·242 grams.
5. 18·715 grams.	**10.** 104·71 litres.	**15.** 1192·702 metres.

Express as grams or metres :—

16. 5 Hg. 7 Dg. 5 g. 7 dg.
17. 5 Hg. 7 g. 8 cg. 5 mg.
18. 6 Kg. 5 g. 7 dg. 8 cg.
19. 7 g. 8 dg. 5 cg.
20. 8 Km. 7 m. 8 dm. 5 mm.
21. 15 Km. 7 m. 8 cm. 9 mm.
22. 15 Hg. 7 g. 8 dg. 9 mg.
23. 17 Kg. 8 Dg. 9 dg. 7 mg.

Express in kilometres :—

24. 7 Km. 8 Hm. 9 Dm.
25. 7 Hm. 5 m.
26. 7 Hm. 5 Dm. 7 dm.
27. 5 Km. 8 Dm 7 dm.
28. 8 Hm. 7 m. 5 cm.
29. 2 Dm. 5 m. 2 cm.
30. 5 Dm. 2 cm. 6 mm.
31. 4 Hm. 5 mm.

MULTIPLICATION

To multiply and divide by powers of 10.

$$7{\cdot}52 \times 10 = (7 \text{ units } 5 \text{ tenths } 2 \text{ hundredths}) \times 10$$
$$= 70 \text{ units } 50 \text{ tenths } 20 \text{ hundredths}$$
$$= 7 \text{ tens } 5 \text{ units } 2 \text{ tenths.}$$
$$= 75{\cdot}2.$$

Thus also $75{\cdot}2 \div 10 = 7{\cdot}52.$

Similarly $7{\cdot}523 \times 100 = 752{\cdot}3$

$752{\cdot}3 \div 100 = 7{\cdot}523$, etc.

Rule.—In order to {**multiply** / **divide**} **a number by 10, 100, 1000, . . . we have to move the figures 1, 2, 3, . . . places to the** {**left** / **right**} **of the decimal point.**

[*If there are not enough figures to do this, 0's must be added either at the end or beginning.*]

Ex. 1. *Multiply* 123·71 *by* 1000.

Product = 123710.

Ex. 2. *Convert* 14·72 metres *into* kilometres.

[This is equivalent to dividing by 1000.]

14·72 metres = ·01472 kilometres.

Ex. 3. *Convert* 182·4 sq. metres *into* sq. dekametres.

[This is equivalent to dividing by 100.]

182·4 sq. metres = 1·824 sq. dekametres.

Ex. 4. *Convert* 1·45728 cu. dekametres *into* cu. metres.

[This is equivalent to multiplying by 1000.]

1·45728 cu. dekametres = 1457·28 cu. metres.

EXERCISE XXXI. (*Oral.*)

What are the values of :—

1. $39{\cdot}37 \times 100$.
2. $3{\cdot}14159 \times 1000$.
3. $2133{\cdot}1416 \div 10000$.
4. $823{\cdot}231 \times 10$.
5. $8372{\cdot}2305 \times 1000$.
6. $2345{\cdot}6781 \times 100000$.
7. $204{\cdot}312 \div 100$.
8. $2043{\cdot}71 \times 1000$.
9. $873{\cdot}112 \times 10000$.
0. $3457{\cdot}117 \times 10$.
1. $8234{\cdot}139 \times 100000$.
2. $325{\cdot}427 \div 100$.
3. $87{\cdot}213 \div 1000$.
4. $227{\cdot}23 \div 100000$.
5. $3{\cdot}14159265 \div 100$.
6. $827{\cdot}325 \times 10000$.
7. $72{\cdot}315 \div 100$.
8. $725{\cdot}078 \div 100000$.
9. $8235{\cdot}92 \div 100$?

Convert :—

0. 18·72 grams. into kilograms.
1. 1·27 kilometres into metres.
2. 41·7 hectolitres into litres.
3. 1·72 sq. metres into sq. dekametres.
4. 45·7241 cu. hectometres into cu. metres.
5. 1427 millimetres into metres.
6. 172 milligrams into kilograms.
7. 1·795 sq. hectometres into sq. decimetres.
8. 82·7 ares into centiares.
9. 14275 centimes into francs.
0. 18926 cu. metres into cu. kilometres.

82 sq. m. 41 sq. dm. 6 sq. cm.
= 82 sq. m. 41·06 sq. dm.
= 82·4106 sq. m.

Conversely, 41·0784 sq. m. = 41 sq. m. 7 sq. dm. 84 sq. cm.

Similarly, 41 cu. m. 127 cu. dm. 400 cu. cm. = 41·127400 cu. m.
= 41·1274 cu. m.

7·0000807 cu. m. = 7·000080700 cu. m. = 7 cu. m. 80 cu. cm. 700 cu. mm.

N.B.—It will be observed that in square measure, the figures to the right of the decimal point are taken 2 at a time, and in cubic measure 3 at a time.

Exercise XXXII.

Write down :—

1. 41 sq. m. 15 sq. dm. 28 sq. cm. in sq. metres.
2. 43 sq. m. 17 sq. cm. 49 sq. mm. „ „
3. 18 sq. m. 5 sq. cm. 70 sq. mm. „ „
4. 14 sq. Hm. 18 sq. m. 14 sq. cm. „ „
5. 82 sq. m. 75 sq. cm. 81 sq. mm. in sq. hectometres.
6. 325 cu. m. 721 cu. dm. 827 cu. cm. in cu. millimetres.
7. 35 sq. m. 72 sq. cm. 81 sq. mm. in sq. dekametres.
8. 25 sq. m. 7 sq. dm. 5 sq. cm. 8 sq. mm. in sq. metres.
9. 251 cu. dm. 731 cu. mm. in cu. metres.
10. 127 cu. cm. 50 cu. mm. „ „

Write down at full length :—

11. 18·725 sq. metres.
12. 4287·8105 cu. metres.
13. 724·178 sq. dm.
14. 1871·41560 cu. dm.
15. 125·782 sq. Km.
16. 1732·045 sq. Hm.
17. 10081·745 cu. metres.
18. 1·827 sq. metres.
19. 10876·42 cu. dm.
20. 4000·008 cu. Dm.

ADDITION AND SUBTRACTION

Ex. 1. *Find the sum of* 5 m. 7 dm. 2 cm. 3 mm., 2 cm. 5 mm., Dm. 5 cm. 2 mm., 2 m. 7 mm., *and express the result in metres.*

5·723 m.
·025
70·052
2·007
77·807 metres = 7 Dm. 7 m. 8 dm. 7 mm.

Ex. 2. *Find the difference between* 72 cu. m. 351 cu. dm. 705 cu. cm *nd* 30 cu. m. 785 cu. cm.

72·351705 cu. m.
30·000785
42·350920 cu. m. = 42 cu. m. 350 cu. dm. 920 cu. cm.

Exercise XXXIII.

Find the sum of (writing the answers at full length) :—

1. 8 m. 7 dm. 2 cm., 5 m. 3 cm. 2 mm., and 9 m. 2 mm.
2. 5 m. 2 dm. 3 mm., 8 cm. 2 mm., 9 dm. 5 cm. 3 mm., and 7 mm.
3. 7 m. 8 mm., 5 dm. 2 cm. 6 mm., 9 cm. 8 mm., and 7 m. 5. dm. 2 cm.
4. 4 m. 7 dm. 2 mm., 5 m. 2 cm. 8 mm., 9 m. 7 dm. 2 mm., and 5 dm. 2 cm. 8 mm.
5. 72 sq. m. 85 sq. dm. 72 sq. cm., 91 sq. m. 32 sq. cm. 81 sq. mm., and 25 sq. dm. 8 sq. cm. 2 sq. mm.
6. 821 cu. m., 517 cu. m. 221 cu. dm. 713 cu. cm., 403 cu. dm. 70 cu. cm., and 2 cu. m. 7 cu. dm. 51 cu. cm.
7. 8 Kg. 5 Hg. 9 Dg. 9 dg. 5 cg., 8 Hg. 5 cg. 7 mg., and 7 Dg. 5 g. 8 dg. 9 cg.
8. 7 g. 5 dg. 6 mg., 8 dg. 7 cg. 5 mg., 7 Dg. 9 g. 8 dg. 7 cg., and 5 dg. 7 cg. 9 mg.
9. 81 sq. m. 7 sq. dm. 5 sq. cm., 92 sq. m. 71 sq. dm. 52 sq. cm., 73 sq. cm. 5 sq. mm., and 2 sq. m. 5 sq. dm. 82 sq. cm.
0. 92 sq. dm. 5 sq. cm. 11 sq. mm., 51 sq. dm. 7 sq. cm., 82 sq. m. 7 sq. mm., and 42 sq. m. 7 sq. dm. 8 sq. mm.

Express in metres the sum of :—

11. 5 Km. 7 Hm. 5 m. 7 cm. 2 mm., 5 Dm. 6 m. 7 dm. 5 cm. 6 mm., 8 Km. 7 dm. 5 mm.

12. 5 Hm. 7 Dm. 6 m. 7 dm. 5 mm., 7 m. 8 dm. 9 cm. 9 mm., 5 Hm., 7 Hm. 5 Dm. 7 dm. 6 mm., 7 m. 5 dm. 2 cm. 3 mm.

13. 7 m. 8 cm. 2 mm., 5 Dm. 7 m. 2 dm. 3 mm., 4 Hm. 7 dm. 5 mm., 8 Km. 3 Dm. 3 dm. 7 cm., 8 Hm. 3 mm.

14. 50 Dm. 7 m. 8 cm., 50 m. 7 dm. 2 mm., 31 m. 2 cm. 9 mm., 52 m. 5 dm. 9 cm. 9 mm., 9 m. 5 dm. 7 cm. 9 mm.

15. 5 m. 9 dm. 5 cm. 6 mm., 8 m. 9 mm., 82 Dm. 7 mm., 52 m. 4 cm. 2 mm., 81 m. 7 cm. 2 mm.

Express in kilometres the sum of :—

16. 5 Km. 3 Hm. 2 Dm. 4 m. 5 mm., 3 Dm. 2 m. 7 dm. 1 cm. 4 mm., and 9 Dm. 3 m. 1 dm. 1 cm. 7 mm.

17. 74 Hm. 2 Dm. 7 m. 1 mm., 8 Hm. 7 Dm. 2 m. 3 dm. 4 mm., 9 Dm. 7 m. 2 dm. 1 cm. 4 mm., and 2 cm. 1 mm.

18. 800 Dm. 3 dm. 1 mm., 9 m. 7 mm., and 8 m. 3 dm. 1 mm.

19. 82 Hm. 5 dm. 7 mm., 5 Dm. 9 m. 8 cm., and 7 Hm. 9 cm. 1 mm.

20. 18 Km. 7 Hm. 1 m. 4 dm. 1 cm. 7 mm., 5 cm. 9 mm., 8 m. 7 cm. 2 mm., and 9 m. 1 dm. 3 cm. 4 mm.

Find the difference between (writing the answers at full length) : —

21. 8 Kg. and 9 Hg. 7 g. 5 dg. 3 cg.

22. 3 m. 7 dm. 8 mm. and 5 m. 2 dm. 8 cm.

23. 32 sq. m. 7 sq. dm. and 49 sq. m. 11 sq. dm. 5 sq. mm.

24. 87 sq. dm. 5 sq. cm. and 73 sq. dm. 57 sq. cm.

25. 5 Km. 7 Dm. 5 m. 7 mm. and 3 Dm. 8 m. 9 mm.

26. 9 Kg. 5 Hg. 7 g. 9 dg. and 10 Kg. 9 g. 8 cg. 5 mg.

27. 7 sq. m. 5 sq. cm. 7 sq. mm. and 9 sq. m. 15 sq. dm. 7 sq. cm.

28. 8 m. 7 dm. 5 cm. 3 mm. and 5 m. 7 cm. 9 mm.

Subtract (writing the answers at full length) :—

29. 72 cu. m. 813 cu. dm. from 95 cu. m. 715 cu. cm.

30. 803 cu. m. 51 cu. cm. from 911 cu. m. 703 cu. dm.

31. 5 sq. m. 7 sq. dm. 52 sq. cm. from 8 sq. m. 91 sq. dm.

32. 72 sq. m. 8 sq. dm. from 85 sq. m. 72 sq. cm.

Find the excess of (writing the answers at full length) :—

33. 8 Kg. 9 Hg. over 7 Kg. 9 g. 8 dg.

34. 7 m. 8 dm. 9 cm. 5 mm. over 5 m. 9 cm. 2 mm.

35. 5 m. 7 cm. over 7 dm. 2 cm. 3 mm.

36. 81 sq. Dm. 9 sq. m. 7 sq. cm. over 53 sq. m. 7 sq. cm.

Express in metres the differences between :—

37. 82 m. 5 dm. 2 cm. 7 mm. and 79 m. 5 dm. 2 cm. 9 mm.

38. 52 m. 3 cm. 1 mm. and 17 m. 9 mm.

39. 78 m. 9 mm. and 5 m. 7 dm. 2 cm. 4 mm.

40. 871 m. 9 dm. 2 cm. 5 mm. and 49 m. 5 dm. 2 cm. 7 mm.

41. 825 m. 7 cm. 6 mm. and 709 m. 9 dm. 2 cm. 9 mm.

42. 501 m. 7 dm. 2 cm. 5 mm. and 400 m. 9 cm.

43. 740 m. 9 mm. and 605 m. 2 cm.

44. 8275 m. 9 cm. 8 mm. and 529 m. 9 dm. 9 cm. 9 mm.

Find the value of :—

45. $3{\cdot}271 + 8{\cdot}026 + 5{\cdot}07$.

46. $32{\cdot}41 + 1{\cdot}027 + 3{\cdot}019$.

47. $84{\cdot}023 + 72{\cdot}3 + 81{\cdot}5$.

48. $72{\cdot}124 + 18{\cdot}02 - 49{\cdot}72$.

49. $182{\cdot}49 + 48{\cdot}56 - 15{\cdot}72$.

50. $100{\cdot}92 + 83{\cdot}45 - 17{\cdot}8$.

MULTIPLICATION BY A DECIMAL

It having been shown that the consecutive digits of a number involving a decimal point bear the same relation to one another as those in an ordinary number, it follows, as has already been pointed out, that the ordinary method of multiplication is applicable to decimals, each partial product starting one column farther to the right than the preceding. It will be convenient (particularly when dealing with contracted work), to arrange so that there is only *one figure to the left of the decimal point* in the multiplier; this may easily be effected by moving the figures the required number of places to the left or right of the decimal point in the multiplier and the same number of places in the *opposite direction* in the multiplicand, for we shall then have multiplied and divided (or *vice versâ*) the product by the same powers of 10, and shall consequently not have altered its value.

Ex. 1. *Multiply* 182·73 *by* ·327.

$$\begin{array}{r} 18{\cdot}273 \\ 3{\cdot}27 \\ \hline 54{\cdot}819 \\ 3{\cdot}6546 \\ 1{\cdot}27911 \\ \hline 59{\cdot}75271 \end{array}$$ Answer.

[*Rough Check:* $18 \times 3 = 54$.]

[In the *first partial product*, we are multiplying by 3 *units*, and therefore there will be the **same number of figures to the right of the decimal point as there are in the multiplicand.**]

Ex. 2. *Multiply* 72·41 *by* 20·03.

$$\begin{array}{r} 724{\cdot}1 \\ 2{\cdot}003 \\ \hline 1448{\cdot}2 \\ 2{\cdot}1723 \\ \hline 1450{\cdot}3723 \end{array}$$ Answer.

[*Rough Check:* $724 \times 2 = 1448$.]

N.B.—If the result is checked, it is practically impossible to misplace the decimal point.

In some cases it will be found convenient to multiply by factors.

Exercise XXXIV.

Multiply :—

1. 5·82 by 8, 12, 32, 24.
2. 51·25 by 5, 10, 15, 20.
3. 62·5 by 3, 6, 12, 15.
4. ·52 by 4, 5, 8, 15.
5. ·525 by 7, 14, 21, 28.
6. ·056 by 8, 16, 6, 18.
7. ·0045 by 9, 12, 18, 27.
8. 51·2 by ·5, ·05, ·005.
9. 7·5 by 1·5, ·15, ·015.
10. 82·5 by 2·5, 3·6, 4·8, 8·4.
11. 712·25 by ·25, 6·25, 3·75.
12. 8·327 by 7·205.
13. 81·508 by 72·32.
14. 315·23 by ·517.
15. 320·03 by ·052.
16. 307·08 by ·00058.
17. 54·965 by ·0072.
18. 3·1416 by 3·071.
19. 323·496 by ·4771.
20. ·30103 by 3·142.
21. 3·14159265 by ·47712.
22. 921·0007 by ·000052.
23. 8761·03 by ·00701.
24. 325·051 by 9·262.
25. 1234·56 by 5·006.
26. 821·8 by 5·0101.
27. ·003625 by 74·2.
28. ·0204 by 40·2.
29. ·008 by ·0016.
30. 4·3072 by ·004372.

Find the value of :—

31. ·421 × 7·26 × 8·15.
32. 1·002 × ·032 × 8·9.
33. 17·21 × 154·2 × 17·35.
34. ·0089 × 16·3 × 25·02.
35. 982·5 × 47·25 × 1·082.

DIVISION

Rule.—Before dividing, move the decimal point the same number of places to the right or left in the divisor and dividend, until there is *one figure to the left of the decimal point* **in the divisor.**

Ex. 1. *Divide* 332·8533 *by* 42·51.

i.e. 33·28533 by 4·251.

[*Rough Check:* Since $33 \div 4 = 8$ (approx.), it follows that *misplacing the decimal point in the quotient should be impossible.*]

Having determined the probable number of figures to the left of the decimal point in the quotient, the working then proceeds exactly as in ordinary long division.

```
      7·83        Answer.
4·251)33·28533
      29 757
       3 5283
       3 4008
         12753
         12753
```

Ex. 2. *Divide* 5·152554 *by* ·5247,

i.e. 51·52554 by 5·247.

[*Rough Check:* Since $51 \div 5 = 10$ (approx.), it follows that there *may be* 2 but *possibly* only 1 figure to the left of the decimal point in the quotient.]

On dividing as in ordinary Long Division (neglecting the decimal points), it is at once obvious that the first figure of the quotient is 9, and consequently there is only 1 figure to the left of the decimal point in the quotient.

```
      9·82          Answer.
5·247)51·52554
      47 223
      ------
       4 3025
       4 1976
       ------
         10494
         10494
         -----
```

Ex. 3. *Divide* 17·963 *by* ·0781,

i.e. 1796·3 by 7·81.

[*Rough Check:* $1796 \div 8 = 224$ (approx.).]

```
       230          Answer.
7·81)1796·3
     1562
     ----
      2343
      2343
      ----
```

[On dividing as in ordinary Long Division, the first digit of the quotient is 2; from the Rough Check, we know that this represents 2 hundreds, and it is consequently placed over the 7 of the dividend. At the next step, the division is exact.

The *Rough Check having been performed first*, there should be no danger of writing the quotient as 23 instead of 230.]

N.B.—*In some cases it may be more convenient to have all the figures to the left of the decimal point in the divisor, and then to divide by factors.*

Ex. 4. *Divide* ·00406 *by* ·056,

i.e. 4·06 by 56.

```
7 | 4·06
8 |  ·58
  ------
   ·0725      Answer.
```

Exercise XXXV. (*Oral.*)

Divide :—

1. 85·75 by 5.
2. 73·86 by 3.
3. 14·64 by 4.
4. 15·168 by 8.
5. 42·49 by 7.
6. 82·95 by 50.
7. 18·44 by 400.
8. 843·39 by 30.
9. 56·497 by 7000.
10. 34·248 by 80.
11. 1·21 by 110.
12. 49·048 by 40, 800, 2000.
13. 12·6 by 2, 3, 50, 6, 700.
14. 8·1 by 30, 5, 600, 90.
15. 31·5 by 50, 7, 900.
16. 3·85 by 5, 700, 110.

Exercise XXXVI.

Divide :—

1. ·00081 by 27.
2. 1·44 by 12 and ·012.
3. 12·6 by ·006 and ·07.
4. 13·08 by 4 and ·008.
5. ·156 by ·012 and 1·3.
6. 13·2 by 1·2 and 11.
7. 172·8 by ·0012 and 4·8.
8. ·000243 by ·0027.
9. 28900 by ·0017.
10. 37·8 by ·035.
11. 28·9 by 17 and ·289 by ·017.
12. ·0216 by ·25, and 1·5 by ·0064.
13. 7·73682 by 101·4.
14. ·03625 by ·29.
15. 384·006 by ·123.
16. 72·5026 by 7·85.
17. 145·0052 by ·00785.
18. ·00307326 by 3·013.
19. ·282892 by 3·94.
20. 210720·6 by ·4206.
21. ·2107206 by 42·06.
22. 12·6 by ·0012.
23. ·065341 by ·00475.
24. ·2332728 by 32·58.
25. ·209784253 by 36·151.
26. 174·13238 by 5·615.
27. 311·010207 by ·076359.
28. 1·13204182 by 24·0604.
29. 2·8210051 by 2933·2.
30. 1102·4649 by 26·063.
31. 157·311 by ·3405.
32. 4·65131 by 596·8.
33. 4·7362 by ·0435 to five places of decimals.
34. ·00729 by ·2735 „ „
35. 183·026 by 2·03 to three „

How many times is the second quantity contained in the first quantity in each of the following examples?—

36. 30 m. 1 dm. 2 cm. *and* 6 mm.

37. 293 Km. 9 Hm. 1 Dm. 6 m. 6 dm. 6 cm. 9 mm. *and* 5 Hm. 4 Dm. 1 m. 2 dm. 8 cm. 3 mm.

38. 31 m. 5 dm. *and* 1 dm. 2 cm. 6 mm.

39. 5 m. 2 dm. *and* 3 dm. 2 cm.

40. 2147 m. 4 cm. *and* 3 cm. 6 mm.

41. 408 m. 9 dm. 8 cm. *and* 2 dm. 8 cm. 6 mm.

42. 3 m. 1 dm. 2 cm. 5 mm. *and* 1 dm. 2 cm. 5 mm.

43. 1 Hm. 8 Dm. 7 m. 5 dm. *and* 7 cm. 5 mm.

44. 44 m. 1 dm. 4 cm. 4 mm. *and* 35 m. 6 dm.

45. 978 m. 5 dm. 1 cm. 6 mm. *and* 32 m.

AREAS AND VOLUMES

By squared paper (or otherwise) it may be shown that—

The **area of a rectangle** = length × breadth,

Area of 4 walls of a room = perimeter × height,

where **perimeter** = 2 (length + breadth).

Volume of rectangular box = length × breadth × height,

$$\therefore \textit{height} = \frac{\text{volume}}{\text{length} \times \text{breadth}}, \text{ etc.}$$

Ex. 1. *Find in sq. cm. the area of the* 4 *walls of a room whose height is* 4 m. 5 dm. 7 cm., *length* 7 m. 8 cm., *and breadth* 6 m. 5 dm. 7 mm.

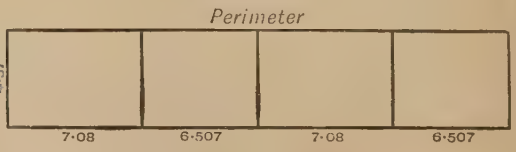
Perimeter = 2 (7·08 + 6·507) m.

= 2 × 13·587 m.

= 27·174 m.

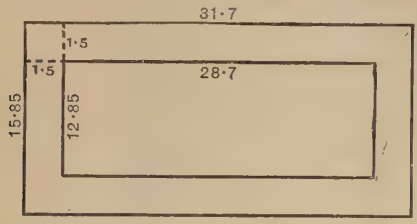
∴ Area of 4 walls = 4·57 × 27·174 sq. m.

27·174
4·57
108·696
13·5870
1·90218
124·18518 sq. m.



∴ Area = 1241851·8 sq. cm.

Ex. 2. *A garden consists of a lawn with a gravel path outside. If the garden is* 31 *m.* 7 *dm. long and* 15 *m.* 8 *dm.* 5 *cm. broad, find the area of the path, its width being* 1 *m.* 5 *dm.*



Area of garden $= 31{\cdot}7 \times 15{\cdot}85$ sq. m.
$= 502{\cdot}445$ sq. m.

Area of lawn $= (31{\cdot}7 - \overline{2 \times 1{\cdot}5})(15{\cdot}85 - \overline{2 \times 1{\cdot}5})$ sq. m.
$= 28{\cdot}7 \times 12{\cdot}85$ sq. m.
$= 368{\cdot}795$ sq. m.

Area of path $= 502{\cdot}445 - 368{\cdot}795$ sq. m.
$= 133{\cdot}65$ sq. m.

158·5
3·17
475·5
15·85
11·095

128·5
2·87
257·0
102·80
8·995

N.B.—The formula $x^2 - y^2 = (x+y)(x-y)$ will frequently be found useful.

Exercise XXXVII.

Find the *areas of the following rectangles* where

1. Length is 13 m. 4 cm., breadth 10 m. 4 dm. 3 cm. Express answer in sq. m.
2. Length is 12 m. 3 mm., breadth 10 m. 5 dm. Express answer in sq. Km.
3. Length is 12 m. 3 cm. 2 mm., breadth 10 m. 2 dm. 3 cm. Express answer in sq. dm.
4. Length is 7 m. 5 dm., breadth 9 m. 3 cm. Express answer in sq. m.
5. Length is 7 m. 5 dm. 2 cm. 6 mm., breadth 5 m. 7 dm. 3 cm. Express answer in sq. dm.
6. Length is 36 m. 7 dm. 2 mm, breadth 12 m. 5 dm. 7 cm. Express answer in sq. Km.

Find the *areas to be papered* in rooms of the following dimensions :—

7. Length = 13 m. 4 cm., breadth = 10 m. 4 dm. 3 cm., height = 13 m 2 cm. Express answer in sq. m.
8. Length = 12 m. 3 mm., breadth = 10 m. 5 dm. 3 cm., height = 12 m 5 cm. Express answer in sq. Km.
9. Length = 13 m. 3 cm. 2 mm., breadth = 10 m. 2 dm. 3 cm., height = 8 m. 2 dm. 4 mm. Express answer in sq. cm.
10. Length = 10 m. 5 dm., breadth = 9 m. 3 cm., height = 7 m. 3 dm Express answer in sq. Km.
11. Length = 13 m. 2 dm. 4 mm., breadth = 10 m. 2 cm. 5 mm., heigh = 7 m. 2 dm. 3 cm. Express answer in sq. dm.
12. Length = 10 m. 9 dm. 5 cm. 3 mm., breadth = 9 m. 2 cm. 3 mm., heigh = 8 m. 3 mm. Express answer in sq. mm.
13. Length = 14 m. 7 dm. 3 cm., breadth = 10 m. 2 dm. 3 cm. 4 mm height = 6 m. 7 dm. 2 cm., when there is a door 3 m. 2 dm. by 1 m 7 dm. and a window 2 m. 3 cm. by 2 m. 1 cm. 2 mm. Give th answer in sq. dm.
14. Length = 11 m. 2 dm. 3 cm., breadth = 10 m. 2 cm. 3 mm., height = 3 m. 7 dm. 2 mm., when there is a door 2 m. by 1 m. 7 dm. and window 2 m. 7 dm. by 1 m. 8 dm. Express answer in sq. dm.

Find the *volumes of rectangular boxes* of the following dimensions :—

15. Length = 5 m. 7 dm., breadth = 4 m. 7 dm. 5 cm., height = 4 m. 5 m Answer in cu. metres.
16. Length = 13 m. 7 dm. 2 cm., breadth = 10 m. 5 dm., height = 9 m 7 dm. 2 cm. Answer in cu. dekametres.
17. Length = 12 m. 7 cm., breadth = 11 m. 5 cm. 2 mm., height = 5 m 7 dm. Answer in cu. decimetres.
18. Length = 8 m. 7 dm. 2 mm., breadth = 7 m. 2 dm. 3 cm., height = 5 m 2 dm. Answer in cu. metres.
19. Length = 7 m. 2 mm., breadth = 6 m. 5 cm., height = 5 m. 2 dm Answer in cu. metres.
20. Length = 11 m. 2 dm. 3 cm., breadth = 10 m. 5 dm., height = 5 m. 2 cm Answer in cu. dekametres.

. Find in cubic metres the content of a block of stone 2 metres long, 2 m. broad, and 1 m. 2 dm. 5 cm. deep.

. Find the area of a grass plot in a garden whose length is 55 m. 7 dm. and breadth 31 m. 7 dm. 5 cm., when there is a path round the grass plot 1 m. 7 dm. broad. Give the answer in sq. metres.

. What is the area to be covered with gravel in a garden whose dimensions are 105 m. 7 dm. 5 cm. by 71 m. 5 dm., when the path running round the lawn is 2 m. 5 dm. broad? Give the answer in sq. Dm.

. If an oblong garden of length 152 m. 7 dm. 5 cm. and breadth 105 m. 2 dm. has paths 2 m. 4 dm. wide running the whole length of the two long sides, what is the area remaining to be turfed? Give the answer in sq. metres.

. A garden of length 113 m. 7 dm. and breadth 52 m. 3 dm. is covered with turf, with the exception of two flower-beds along the shorter sides of breadth 2 m. 5 dm., and two oblong flower-beds near the middle of length 10 m. 5 dm. and breadth 2 m. 5 dm. Express in sq. dm. the amount to be turfed.

. A garden of length 59 m. 7 dm. and breadth 52 m. 7 dm. 5 cm. consists of a lawn 53 m. 3 dm. by 48 m. 3 dm. 5 cm. and a gravel path all round. Express in sq. m. the area of the path.

. The area of a rectangle is 184 sq. m. 96 sq. dm., and its length is 51 m. 2 dm.; what is its breadth in metres?

. If the area of a rectangle is 4000 sq. m., and its breadth 3 m. 1 dm. 2 cm. 5 mm., determine its length in dekametres.

. Given that the area of a field is 1406 sq. Km. 27 sq. Hm. 75 sq. Dm., what will be its length expressed in kilometres if the breadth is 33·05 kilometres?

. Given that a room is 11·25 m. long, 9 m. broad, 4·5 m. high, and that it is papered with paper 7·5 dm. wide, find in metres the length of paper required.

. Given that a room is 1·05 Dm. long, ·825 Dm. broad, and 4 m. high, what is the length of paper required for the walls, if the breadth of the paper is 5 dm.? Express the answer in hectometres.

Miscellaneous Problems on the Metric System

Exercise XXXVIII.

1. 2·87 kilograms of mercury are in a jar weighing 0·245 kilogram What does the whole weigh?

2. A cyclist rides 16 kilometres in an hour. How many minutes an seconds is this for 1 kilometre?

3. How many pieces of wood of 3·4 decimetres can be cut from a pie 5 metres long, and what will be the length in decimetres of the pie left over?

4. What is the cost in francs and centimes of 18 kilograms of butter at francs 15 centimes per kilogram?

5. Find the value of 0·8259 of 2 kilometres in kilometres, metres, and th decimal of a metre.

6. A train travels at the rate of 55 kilometres an hour. Find, to th nearest metre, how far it goes in a minute.

7. Into how many packets, each containing 3·54 kilograms, can 959·3 kilograms be divided?

8. A train does a journey of 67·2 kilometres in 1 hour 10 minutes. Fin its average speed in centimetres per second.

9. A rectangular field is 154·5 metres long and 103·7 metres wide. Fin its value at 2·7 francs per sq. metre.

10. A piece of land is 7 kilometres 5 Hm. long and 4 kilometres broa What is its area in ares?

11. The length of one side of a rectangle is 1·92 kilometres and of the oth 1·35 kilometres. Find the area in hectares.

12. The area of a field is 242 hectares 82 ares, and the length is 1·71 ki metres. What is the breadth in kilometres?

3. The length of a rectangle is 2·37 dekametres and the area 336·54 centiares. Calculate the breadth in dekametres.

4. Some property which is 4287 metres long and 3420 metres broad, sells at 600 francs per hectare. What is the total selling price?

5. What will be the cost of 8 hectolitres 72 litres at 140·8 francs per hectolitre?

6. What will be the weight of 5 metres 75 centimetres of cloth, if 10 metres of it weigh 2 kilograms?

7. Find the volume (in litres) of a block of stone 1 metre long, 90 centimetres wide, and 15 centimetres thick.

8. Find the value of 7 litres 5 decilitres of wine at 1 franc 40 centimes per litre.

9. If the volume of a rectangular block of stone is 15·54 cu. metres and the area of the base 8 sq. m. 88 sq. dms., find the height in metres.

10. Three barrels contain 3·27 hectolitres, 5·92 litres, 115·72 litres of beer. How many litres do they contain altogether?

. A vessel filled with water weighs 74·52 kilograms, and when empty weighs 12·72 kilograms. How many grams of water will it hold?

. On a debt of 175 francs, amounts of 10 francs 20 centimes, 41 francs, and 17 francs 35 centimes have been paid. How much remains?

. What is the cost of 15·5 litres of wine at 55 centimes per litre?

. If 15 centilitres of oil per hour are required for a lamp, how long will 90 litres last?

. What do 2 hectolitres 50 litres of milk cost at 30 centimes per litre?

. Find the cost of 55 yards of cloth at 2 dollars 25 cents per yard.

. What is the cost of 72 metres of matting at 2 francs 32 centimes a metre?

. Express ·0583 kilometres in metres and centimetres.

29. How many cubic metres are there in a block 3 metres long, 1·5 deci metres broad, and ·7 decimetres high?

30. If a train travels at the rate of 95 kilometres per hour, how many hours will it take to go a journey of 535 kilometres? (Answer to 2 places of decimals.)

31. A person buys 15 articles at 5 francs 25 centimes each, and 12 a 3·4 francs each, he then has 10 francs left. How much had he to start with?

32. A solid piece of brickwork is composed of 9850 bricks; the averag volume of each brick with its share of the mortar is 1·5 cu. deci metres. Find the total volume in litres.

33. Find the cost of 1255 articles at 2 dollars 20 cents each.

34. A man buys 45 kilograms of goods at 1 franc 30 c. per kilogram and sells them at 1 franc 75 c. per kilogram. What is his profit?

35. A man buys 51·3 metres of cloth at 3·7 francs per metre, 29·4 metre at 4·5 francs per metre, and sells the whole at 400 francs. What does he gain?

36. If 13·25 Km. of a railway 108 Km. long have been built, what distance remains to be built?

37. A cistern measures 2·75 m. in length, 2·8 m. in breadth and 1·7 m. i depth. How many litres will it hold?

38. If ribbon costs 1 f. 8 c. per metre, how much can be bought fo 17 f. 55 c.?

VULGAR FRACTIONS

$\frac{a}{b}$ means that the unit is to be divided into b parts and a of these parts ken.

Ex. 1. *Find the value of* $\frac{1}{3}$ *of* £2, 2s. 9d.

$$3\,|\,\underline{£2\ \ 2\ \ 9}$$
$$14\ \ 3 \quad \textbf{Answer.}$$

Ex. 2. *Find the value of* $\frac{2}{5}$ *of* £2, 5s. 10d.

[Here we have to divide by 5 and multiply by 2; it will probably be and more convenient to multiply first.]

$$£2\ \ 5\ 10$$
$$2$$
$$5\,|\,\underline{4\ 11\ \ 8}$$
$$18\ \ 4 \quad \textbf{Answer.}$$

[*Rough Check:* $\frac{2}{5}$ of £2, 5s. 0d = 18s.]

Exercise XXXIX.

Find the value of :—

1. $\frac{1}{4}$ of £4, 4s. 8d.
2. $\frac{1}{5}$ of £11, 5s. 5d.
3. $\frac{1}{4}$ of £43, 5s. 1d.
4. $\frac{1}{6}$ of £92, 6s.
5. $\frac{1}{11}$ of £82, 3s. 7d.
6. $\frac{1}{6}$ of £25, 3s. 6d.
7. $\frac{1}{13}$ of £42, 7s. 2d.
8. $\frac{1}{8}$ of £130, 7s. 9d.
9. $\frac{1}{10}$ of £125, 5s. 10d.
10. $\frac{1}{11}$ of £227, 5s. 9d.
11. $\frac{7}{16}$ of 2s. 6d.
12. $\frac{5}{8}$ of £27, 13s. 4d.
13. $\frac{1}{4}$ of £43, 3s. 4d.
14. $\frac{2}{6}$ of £27, 7s. 1d
15. $\frac{5}{7}$ of £22, 15s.
16. $\frac{3}{4}$ of £93, 17s. 4d.
17. $\frac{7}{9}$ of £221, 13s. 3d.
18. $\frac{3}{11}$ of £402, 3s. 9d.
19. $\frac{7}{16}$ of £95, 12s.
20. $\frac{3}{4}$ of 7 ozs. 5 dwts.
21. $\frac{5}{6}$ of 2 ozs. 15 dwts. 6 grs.
22. $\frac{7}{8}$ of 3 lbs. 5 ozs. 10 dwts. 8 grs.
23. $\frac{8}{11}$ of 1 lb. 11 ozs. 14 dwts. 9 grs.
24. $\frac{10}{13}$ of 39 lbs. 3 ozs. 5 dwts.
25. $\frac{5}{9}$ of 7 m. 2 dms. 3 cms.

Reduction to Lowest Terms

The value of a fraction is not altered by multiplying or dividing the numerator and denominator by the same quantity,

$$i.e.\ \frac{a}{b} = \frac{am}{bm}.$$

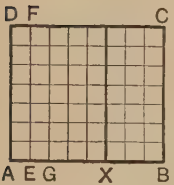
As we have already seen, $\frac{a}{b}$ means that each unit has been divided into b equal parts and a of them taken.

If each of the b parts be subdivided into m smaller parts, and m times as many (*i.e.* am) of them taken as on the previous occasion, we shall obviously have the same result, the operations being expressed by $\frac{am}{bm}$.

$$\text{Thus } \frac{a}{b} = \frac{am}{bm}.$$

This result may be represented Geometrically, in particular cases, by taking the rectangle $ABCD$ to represent the unit; let it be divided into 8 columns DE, FG, etc., of equal breadth, and then take 5 of these columns so as to make up the rectangle DX.

$$DX \text{ represents } \frac{5}{8}.$$

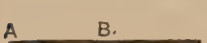


If now the rectangle be further split up into 7 rows, it will be divided altogether into 8×7 small rectangles, of which the figure DX will include 5×7.

Thus DX also represents $\frac{5 \times 7}{8 \times 7}$.

$$\therefore \quad \frac{5}{8} = \frac{5 \times 7}{8 \times 7}.$$

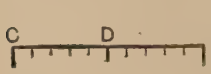
The same result may be shown in certain cases by *comparing the different divisions on a ruler*, when it is at once obvious that



$AB = 4$ eighths of an inch $= 1$ half of an inch,

$$i.e. \quad \frac{4}{8} = \frac{1}{2};$$

also



$CD = 5$ tenths of an inch $= 1$ half of an inch,

$$i.e. \quad \frac{5}{10} = \frac{1}{2}.$$

Ex. 1. *How many sixteenths is* $\frac{3}{8}$?

$$\frac{3}{8} = \frac{3 \times 2}{8 \times 2} = \frac{6}{16}.$$

Ex. 2. *Reduce* $\frac{567}{1323}$ *to its lowest terms.*

$\frac{567}{1323} = \frac{63}{147}$ *(dividing numerator and denominator by 9).*

$= \frac{21}{49}$ („ „ „ 3).

$= \frac{3}{7}$ („ „ „ 7).

If there are no small factors, it will be necessary to find the H.C.F.

Ex. 3. *Reduce* $\frac{1817}{4661}$ *to its lowest terms.*

1	1817	4661	2
	1027	3634	
3	790	1027	1
	711	790	
	79	237	3
		237	

$$\begin{array}{r} 23 \\ 79)1817 \\ 158 \\ \hline 237 \\ 237 \\ \hline \end{array} \qquad \begin{array}{r} 59 \\ 79)4661 \\ 395 \\ \hline 711 \\ 711 \\ \hline \end{array}$$

$$\therefore \frac{1817}{4661} = \frac{79 \times 23}{79 \times 59} = \frac{23}{59}.$$

Exercise XL.

[*Examples 1 to 6 should be taken orally.*]

1. How many sixteenths are there in $\frac{1}{2}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{5}{8}$?
2. „ „ eighths „ $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$?
3. „ „ quarters „ $\frac{12}{16}$, $\frac{2}{12}$, $\frac{10}{20}$, $\frac{18}{24}$, $\frac{27}{36}$?

Reduce to their lowest terms :—

4. $\frac{14}{21}$, $\frac{21}{35}$, $\frac{36}{42}$, $\frac{35}{50}$, $\frac{36}{45}$, $\frac{24}{36}$, $\frac{72}{81}$, $\frac{45}{60}$, $\frac{54}{91}$.
5. $\frac{32}{80}$, $\frac{59}{73}$, $\frac{42}{72}$, $\frac{22}{55}$, $\frac{27}{54}$, $\frac{30}{45}$, $\frac{51}{62}$, $\frac{49}{56}$, $\frac{35}{55}$.
6. $\frac{24}{85}$, $\frac{36}{82}$, $\frac{38}{95}$, $\frac{48}{96}$, $\frac{75}{106}$, $\frac{54}{72}$, $\frac{36}{84}$, $\frac{69}{75}$, $\frac{64}{80}$.
7. $\frac{202}{515}$, $\frac{115}{185}$.
8. $\frac{455}{1015}$, $\frac{132}{840}$.
9. $\frac{1205}{2736}$, $\frac{405}{1272}$.
10. $\frac{1155}{1925}$, $\frac{4248}{5256}$.
11. $\frac{272}{360}$, $\frac{144}{210}$.
12. $\frac{385}{595}$, $\frac{2016}{2079}$.
13. $\frac{1261}{1645}$, $\frac{285}{551}$.
14. $\frac{1244}{2502}$, $\frac{1155}{1764}$.
15. $\frac{1056}{2664}$.
16. $\frac{4872}{9135}$.
17. $\frac{5148}{17028}$.
18. $\frac{126}{551}$.

Arrangement in Order of Magnitude

Ex. *Arrange in ascending order of magnitude* $\frac{11}{14}, \frac{15}{28}, \frac{19}{35}$.

The L.C.M. of 14, 28, 35 is 140,

$\therefore \frac{11}{14}, \frac{15}{28}, \frac{19}{35} = \frac{110}{140}, \frac{75}{140}, \frac{76}{140}$ respectively.

$\therefore$ Order of magnitude is $\frac{15}{28}, \frac{19}{35}, \frac{11}{14}$.

Exercise XLa.

Arrange in ascending order of magnitude :—

1. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{8}$.
2. $\frac{4}{5}, \frac{9}{10}, \frac{7}{8}, \frac{13}{14}$.
3. $\frac{2}{3}, \frac{9}{13}, \frac{5}{9}, \frac{7}{11}$.
4. $\frac{9}{14}, \frac{6}{11}, \frac{8}{13}, \frac{7}{12}$
5. $\frac{5}{12}, \frac{2}{9}, \frac{1}{2}, \frac{3}{10}$.

Arrange in descending order of magnitude :—

6. $\frac{3}{5}, \frac{5}{11}, \frac{7}{13}, \frac{4}{7}$.
7. $\frac{1}{3}, \frac{4}{9}, \frac{9}{29}, \frac{7}{27}$.
8. $\frac{3}{10}, \frac{1}{5}, \frac{4}{11}, \frac{3}{23}$.
9. $\frac{5}{8}, \frac{17}{20}, \frac{19}{30}, \frac{23}{32}$.
10. $\frac{3}{8}, \frac{5}{24}, \frac{7}{36}, \frac{4}{15}$.
11. $\frac{7}{9}, \frac{10}{33}, \frac{17}{21}, \frac{13}{27}$.
12. $\frac{8}{9}, \frac{3}{11}, \frac{7}{18}, \frac{9}{22}, \frac{5}{36}$.

Mixed Numbers and Improper Fractions

A *Mixed Number* is the sum of a whole number and a fraction.

In just the same way that—

5 marbles + 6 marbles = 11 marbles,

so, 5 twelfths + 6 twelfths = 11 twelfths.

i.e. $\frac{5}{12} + \frac{6}{12} = \frac{11}{12} = \frac{5+6}{12}$.

Ex. 1. $\frac{7}{6} = \frac{6+1}{6} = \frac{6}{6} + \frac{1}{6} = 1 + \frac{1}{6} = 1\frac{1}{6}$.

Thus to reduce an Improper Fraction to a Mixed Number, *divide the denominator into the numerator; the quotient gives the whole number, and the remainder the numerator of the resulting proper fraction.*

Ex. 2. $5\frac{2}{5} = 5 + \frac{2}{5} = \frac{40}{8} + \frac{2}{5}$ (for since 8 eighths = 1, ∴ 40 eighths = 5)

$= \frac{42}{8}$.

Ex. 3. *Reduce* $\frac{245}{18}$ *to a mixed number.*

$$\begin{array}{r|l} 3 & 245 \\ 6 & 81 \\ \hline & 13 \end{array} \qquad \begin{array}{l} 2 \\ 3 \end{array} \Big\} 11$$

$\therefore \frac{245}{18} = 13\frac{11}{18}$. Answer.

Ex. 4. *Reduce* $14\frac{8}{95}$ *to an improper fraction.*

$$\begin{array}{r} 95 \\ 2 \\ \hline 190 \\ 7 \\ \hline 1338 \end{array}$$

$\therefore 14\frac{8}{95} = \frac{1338}{95}$. Answer.

Exercise XLI.

[1 *to* 6 *should be taken orally.*]

Express as improper fractions :—

1. $1\frac{3}{4}$, $1\frac{7}{16}$, $4\frac{1}{3}$, $8\frac{2}{5}$, $7\frac{4}{11}$, $1\frac{9}{11}$, $3\frac{2}{5}$, $4\frac{2}{3}$.

2. $3\frac{1}{2}$, $7\frac{2}{3}$, $8\frac{1}{4}$, $5\frac{2}{7}$, $4\frac{1}{5}$, $6\frac{2}{7}$, $4\frac{5}{6}$, $8\frac{8}{9}$.

3. $1\frac{3}{7}$, $2\frac{2}{3}$, $4\frac{1}{2}$, $7\frac{1}{5}$, $8\frac{5}{7}$, $9\frac{2}{11}$, $10\frac{8}{9}$, $7\frac{4}{11}$.

Express as mixed numbers :—

4. $\frac{17}{7}$, $\frac{15}{4}$, $\frac{7}{3}$, $\frac{11}{3}$, $\frac{11}{6}$, $\frac{13}{4}$, $\frac{73}{10}$, $\frac{42}{11}$.

5. $\frac{15}{7}$, $\frac{22}{9}$, $\frac{41}{8}$, $\frac{45}{11}$, $\frac{37}{9}$, $\frac{55}{6}$, $\frac{62}{7}$, $\frac{77}{10}$.

6. $\frac{43}{5}$, $\frac{37}{4}$, $\frac{54}{7}$, $\frac{52}{9}$, $\frac{81}{11}$, $\frac{73}{6}$, $\frac{49}{8}$, $\frac{79}{8}$.

Express the following fractions either as whole numbers or mixed numbers in their lowest terms :—

7. $\frac{47}{5}$, $\frac{19}{13}$, $\frac{17}{5}$.

8. $\frac{720}{351}$, $\frac{254}{25}$, $\frac{217}{84}$.

9. $\frac{443}{12}$, $\frac{648}{108}$, $\frac{5876}{157}$.

10. $\frac{2570}{391}$, $\frac{227}{13}$, $\frac{2134}{17}$.

11. $\frac{523}{39}$, $\frac{327}{47}$, $\frac{1341}{45}$.

Express the following mixed numbers as improper fractions :—

12. $7\frac{1}{5}$, $9\frac{2}{7}$, $11\frac{5}{12}$.

13. $8\frac{2}{5}$, $43\frac{7}{11}$, $141\frac{3}{15}$.

14. $13\frac{83}{84}$, $10\frac{16}{21}$, $15\frac{7}{13}$.

15. $17\frac{3}{11}$, $15\frac{7}{19}$, $49\frac{11}{17}$.

Addition and Subtraction

We have seen that $\frac{5}{12} + \frac{6}{12} = \frac{11}{12}$, thus to add fractions it will merely be necessary to reduce them to new fractions having a common denominator.

Ex. 1. *Find the sum of* $3\frac{2}{3}$, $2\frac{5}{9}$, $\frac{4}{27}$.

$$3\tfrac{2}{3} + 2\tfrac{5}{9} + \tfrac{4}{27} = 3\tfrac{18}{27} + 2\tfrac{15}{27} + \tfrac{4}{27} = 5\frac{18+15+4}{27} = 5\tfrac{37}{27} = 5\frac{27+10}{27} = 6\tfrac{10}{27}.$$ Answer.

Ex. 2. *Subtract* $3\frac{2}{3}$ *from* $4\frac{9}{10}$.

$$4\tfrac{9}{10} - 3\tfrac{2}{3} = 1\frac{27-20}{30} = 1\tfrac{7}{30}.$$ Answer.

Ex. 3. *Subtract* $5\frac{4}{7}$ *from* $9\frac{2}{5}$.

$$9\tfrac{2}{5} - 5\tfrac{4}{7} = 4\frac{14-20}{35} = 3\frac{35+14-20}{35} = 3\tfrac{29}{35}.$$ Answer.

Ex. 4. *Simplify* $5\frac{1}{3} + 2\frac{3}{7} - 3\frac{1}{6}$.

$$5\tfrac{1}{3} + 2\tfrac{3}{7} - 3\tfrac{1}{6} = 4\frac{14+18-7}{42} = 4\tfrac{25}{42}.$$ Answer.

Exercise XLII.

[1 *to* 25 *should be taken orally.*]

1. Add $\frac{5}{8}$ and $\frac{2}{8}$, 3 fifths and 1 fifth, $\frac{3}{12}$ and $\frac{5}{12}$, $\frac{2}{6}$ and $\frac{1}{6}$, 5 ninths and 2 ninths.
2. Add $2\frac{1}{2}$ and $\frac{1}{4}$, $1\frac{1}{4}$ and $\frac{1}{8}$, $2\frac{1}{8}$ and $1\frac{1}{16}$, $1\frac{1}{3}$ and $2\frac{2}{3}$.
3. Subtract $\frac{1}{8}$ from $\frac{6}{8}$, $\frac{2}{12}$ from $\frac{4}{12}$, 3 tenths from 7 tenths, $\frac{3}{11}$ from $\frac{7}{11}$.
4. Subtract $1\frac{1}{12}$ from $2\frac{5}{12}$, $2\frac{1}{4}$ from $4\frac{3}{4}$, $1\frac{1}{8}$ from $2\frac{5}{8}$, $1\frac{1}{3}$ from $2\frac{2}{3}$.

Find the value of :—

5. $\frac{1}{8}+\frac{3}{8}+\frac{5}{8}$.
6. $\frac{2}{3}+\frac{5}{3}+\frac{1}{3}$.
7. $\frac{1}{11}+\frac{3}{11}+\frac{10}{11}$.
8. $1\frac{7}{12}-\frac{5}{12}$.
9. $1\frac{15}{17}-\frac{11}{17}$.
10. $1\frac{13}{15}+\frac{17}{15}$.
11. $\frac{3}{8}+\frac{7}{8}+2\frac{1}{8}$.
12. $\frac{3}{7}+\frac{2}{7}+1\frac{5}{7}$.
13. $3\frac{2}{11}-2\frac{1}{11}$.
14. $4\frac{3}{8}-2\frac{1}{4}$.
15. $8\frac{1}{2}+7\frac{2}{3}$.
16. $4\frac{1}{2}-2$.
17. $\frac{5}{7}+\frac{8}{7}-\frac{3}{7}$.
18. $\frac{3}{13}+\frac{8}{13}-\frac{2}{13}$.
19. $11\frac{7}{8}-10\frac{2}{8}$.
20. $4\frac{3}{4}-2\frac{1}{2}$.
21. $\frac{7}{8}+\frac{5}{8}-1$.
22. $\frac{4}{15}+1\frac{1}{15}-\frac{3}{5}$.
23. $2\frac{7}{9}-1\frac{1}{3}$.
24. $\frac{1}{8}+\frac{7}{8}-\frac{3}{4}$.
25. $\frac{2}{3}+4\frac{1}{3}-3$.

Find the sum of :—

26. $7\frac{33}{40}$, $5\frac{5}{16}$.
27. $3\frac{2}{7}$, $5\frac{13}{42}$.
28. $16\frac{1}{7}$, $2\frac{15}{56}$, $1\frac{1}{48}$, $\frac{17}{35}$.
29. $7\frac{11}{18}$, $\frac{17}{24}$, $5\frac{4}{9}$, $11\frac{11}{36}$.
30. $4\frac{7}{20}$, $3\frac{8}{15}$, $1\frac{5}{12}$.
31. $3\frac{11}{16}$, $2\frac{17}{48}$, $1\frac{37}{80}$.
32. $3\frac{7}{12}$, $\frac{11}{18}$, $1\frac{23}{80}$, $4\frac{5}{36}$.
33. $3\frac{5}{7}$, $2\frac{4}{15}$, $1\frac{9}{35}$.
34. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{19}{36}$.
35. $\frac{13}{15}$, $\frac{17}{20}$, $\frac{19}{21}$, $\frac{23}{25}$, $\frac{29}{30}$.
36. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$.
37. $2\frac{1}{15}$, $3\frac{1}{8}$, $\frac{1}{20}$, $1\frac{1}{24}$.
38. $5\frac{9}{16}$, $3\frac{5}{11}$, $\frac{8}{33}$, $2\frac{1}{4}$.
39. $2\frac{1}{4}$, $3\frac{3}{5}$, $1\frac{11}{12}$, $\frac{3}{18}$, $3\frac{1}{25}$.
40. $\frac{4}{5}$, $\frac{7}{10}$, $\frac{4}{7}$, $\frac{2}{21}$.
41. $\frac{2}{3}$, $1\frac{5}{7}$, $2\frac{2}{11}$, $4\frac{7}{9}$.
42. $3\frac{1}{2}$, $5\frac{5}{68}$, $3\frac{3}{16}$, $7\frac{2}{51}$.
43. $4\frac{1}{5}$, $3\frac{3}{10}$, $\frac{7}{30}$, $\frac{4}{35}$.
44. $10\frac{1}{4}$, $3\frac{3}{17}$, $5\frac{2}{25}$, $\frac{8}{15}$.
45. $5\frac{1}{7}$, $3\frac{1}{21}$, $5\frac{3}{49}$, $\frac{5}{28}$.
46. $\frac{3}{11}$, $\frac{5}{42}$, $1\frac{3}{28}$, $5\frac{5}{22}$.
47. $10\frac{3}{7}$, $5\frac{2}{5}$, $4\frac{1}{4}$, $3\frac{1}{3}$.
48. $5\frac{1}{5}$, $3\frac{2}{3}$, $10\frac{2}{21}$, $15\frac{4}{63}$.

Subtract :—

49. $3\frac{5}{8}$ from $6\frac{7}{68}$.
50. $1\frac{33}{128}$ from $8\frac{1}{8}$.
51. $\frac{15}{16}$ from $9\frac{7}{48}$.
52. $5\frac{8}{27}$ from $7\frac{5}{72}$.
53. $25\frac{17}{19}$ from $31\frac{11}{76}$.
54. $7\frac{10}{13}$ from $11\frac{11}{78}$.
55. $3\frac{11}{16}$ from $8\frac{5}{24}$.
56. $1\frac{1}{3}$ from $2\frac{3}{5}$.
57. $30\frac{5}{12}$ from 42.
58. $2\frac{13}{24}$ from $3\frac{17}{32}$.
59. $3\frac{1}{15}$ from $4\frac{7}{30}$.
60. $8\frac{2}{3}$ from $9\frac{1}{15}$.
61. $3\frac{2}{5}$ from $4\frac{1}{7}$.
62. $5\frac{11}{12}$ from $9\frac{1}{5}$.

Find the excess of :—

63. $3\frac{1}{3}$ over $2\frac{1}{2}$.

64. $9\frac{1}{12}$ over $8\frac{1}{3}$.

65. $42\frac{13}{15}$ over $39\frac{12}{25}$.

66. $51\frac{7}{9}$ over $22\frac{17}{21}$.

67. $4\frac{10}{17}$ over $1\frac{4}{5}$.

68. $8\frac{7}{85}$ over $\frac{14}{39}$.

69. $9\frac{4}{53}$ over $7\frac{3}{4}$.

70. $7\frac{11}{32}\frac{9}{0}$ over $3\frac{5}{16}$.

71. $8\frac{8}{38}\frac{8}{5}$ over $4\frac{2}{5}$.

72. $11\frac{7}{1}\frac{7}{32}$ over $5\frac{11}{12}$.

Find the value of :—

73. $\frac{3}{8}-\frac{5}{14}+\frac{1}{20}$.

74. $\frac{2}{3}+\frac{4}{5}+\frac{6}{7}-\frac{8}{9}$.

75. $\frac{3}{20}+\frac{5}{12}+\frac{4}{15}-\frac{5}{6}$.

76. $\frac{2}{3}+\frac{5}{8}+\frac{1}{16}-1\frac{1}{6}$.

77. $4\frac{5}{9}+\frac{3}{7}-\frac{1}{3}$.

78. $3\frac{10}{11}+5\frac{7}{15}-2\frac{9}{22}-4\frac{9}{10}$.

79. $5\frac{1}{2}-7\frac{1}{8}+4\frac{1}{4}+3-\frac{2}{3}$.

80. $\frac{5}{12}-\frac{7}{16}-\frac{1}{8}+\frac{13}{14}-\frac{9}{48}-\frac{5}{42}$.

81. $5\frac{2}{3}-6\frac{3}{4}+7\frac{4}{5}-5\frac{7}{10}$.

82. $\frac{11}{24}-\frac{5}{12}+\frac{2}{5}-\frac{1}{3}$.

83. $3\frac{1}{3}+2\frac{3}{7}-6\frac{8}{9}+1\frac{8}{63}$.

84. $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$.

85. $\frac{1}{2}-\frac{1}{3}-\frac{1}{5}+\frac{7}{10}$.

86. $1\frac{1}{2}+2\frac{1}{12}-3\frac{19}{24}+3\frac{3}{16}$.

87. $1\frac{32}{16}\frac{2}{9}-\frac{14}{65}+\frac{35}{91}+\frac{7}{39}-\frac{21}{52}$.

88. $\frac{4}{7}-2\frac{3}{5}+5-\frac{5}{32}$.

89. $3\frac{2}{7}+5\frac{2}{3}-3+8\frac{5}{9}$.

90. $4\frac{7}{12}-3\frac{9}{10}+\frac{4}{5}$.

91. $4\frac{1}{9}+7\frac{1}{5}+3\frac{11}{45}$.

92. $3\frac{1}{2}-1\frac{7}{8}+4\frac{5}{12}-2\frac{11}{30}$.

93. $8\frac{1}{3}-2\frac{2}{5}-3\frac{7}{10}$.

94. $3\frac{1}{2}+2\frac{1}{12}+3\frac{5}{16}-7\frac{19}{24}$.

95. $3\frac{1}{5}+2\frac{3}{10}+5\frac{11}{15}+6\frac{7}{20}$.

96. $3\frac{1}{2}-4\frac{1}{3}+5\frac{1}{4}-6\frac{1}{5}+7\frac{1}{6}$.

97. $17\frac{1}{2}+12\frac{5}{6}+9\frac{1}{3}$.

Multiplication by Whole Numbers

$\frac{3}{5} \times 4$ means that firstly the unit has been divided into 5 equal parts, and 3 such parts taken, and afterwards that 4 times as many parts taken, *i.e.* 3×4 parts altogether;

$$\therefore \frac{3}{5} \times 4 = \frac{3 \times 4}{5} = \frac{12}{5}.$$

This result may be illustrated geometrically, by taking the rectangle *ABCD* to represent the original unit; this is divided into five equal strips, so that *AE* represents $\frac{3}{5}$. We now have to take a rectangle 4 times as large, *i.e.* *AF*; but this rectangle contains 12 strips and thus represents $\frac{12}{5}$.

Ex. 1. $5\frac{4}{21} \times 7 = \frac{109}{21} \times 7 = \frac{109 \times \cancel{7}}{\cancel{21}_{3}} = \frac{109}{3} = 36\frac{1}{3}$. Answer.

Exercise XLIII.

[1 *to* 10 *should be taken orally.*]

Multiply:—

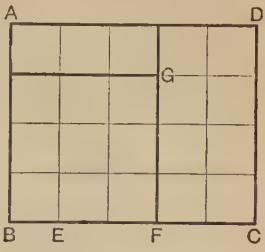
1. $\frac{1}{4}$ by 8, $\frac{2}{5}$ by 4, $\frac{1}{3}$ by 7, $1\frac{2}{5}$ by 9, $1\frac{1}{3}$ by 4, $2\frac{2}{5}$ by 10.
2. $2\frac{1}{4}$ by 7, $1\frac{1}{3}$ by 5, $2\frac{3}{4}$ by 9, $1\frac{1}{5}$ by 12, $2\frac{1}{3}$ by 6, $4\frac{1}{2}$ by 4.
3. $\frac{7}{10}$ by 3, 4 and 5.
4. $\frac{5}{32}$ by 4, 8 and 9.
5. $\frac{3}{25}$ by 5, 10 and 15.
6. $1\frac{5}{8}$ by 4, 5 and 6.
7. $2\frac{2}{3}$ by 3, 6 and 8.
8. $3\frac{2}{3}$ by 3, 6 and 12.
9. $4\frac{2}{5}$ by 5, 6 and 10.
10. $4\frac{1}{4}$ by 2, 5 and 8.
11. $2\frac{11}{10}$ by 3, 5 and 7.
12. $\frac{17}{504}$ by 7, 8 and 9.
13. $5\frac{4}{33}$ by 3, 11 and 22.
14. $4\frac{3}{34}$ by 2, 17 and 5.
15. $7\frac{6}{143}$ by 4, 11 and 13.
16. $5\frac{10}{121}$ by 11, 121 and 20.
17. $4\frac{5}{132}$ by 4, 33 and 12.
18. $2\frac{11}{196}$ by 8, 28 and 98.
19. $\frac{37}{100}$ by 20, 50 and 25.
20. $\frac{41}{94}$ by 4 and 47.
21. $\frac{4}{51}$ by 17 and 30.
22. $1\frac{5}{69}$ by 23 and 15.
23. $3\frac{4}{119}$ by 2 and 19.
24. $\frac{5}{47}$ by 5 and 47.
25. $3\frac{5}{42}$ by 12 and 14.
26. $5\frac{11}{56}$ by 14 and 24.
27. $7\frac{13}{108}$ by 9 and 36.

Division by Whole Numbers

$\frac{3}{5} \div 4$ means that we are to take $\frac{1}{4}$ of $\frac{3}{5}$, in just the same way that $\frac{1}{3}$ of £6 means £6 ÷ 3.

If each of the five parts into which the unit is divided is further subdivided into 4 equal parts, and we take 3 of these new parts, we shall then have $\frac{1}{4}$ of $\frac{3}{5}$; but each of these new parts is derived from the unit by splitting it up into 5×4 or 20 parts.

$$\therefore \frac{3}{5} \div 4 = \frac{1}{4} \text{ of } \frac{3}{5} = \frac{3}{5 \times 4} = \frac{3}{20}.$$



Geometrically we may take $ABCD$ to represent the unit, and divide it into 5 strips such as AE. Then

$\frac{3}{5}$ is represented by AF.

Dividing the strips into 4 equal parts by lines parallel to AD, it follows that

AG will represent $\frac{3}{5} \div 4$.

But the whole figure is now divided into 20 or 5×4 parts, of which AG contains 3, thus

AG also represents $\frac{3}{5 \times 4}$.

$$\therefore \frac{3}{5} \div 4 = \frac{3}{5 \times 4}.$$

Ex. *Divide* $3\frac{4}{7}$ *by* 5.

$$3\frac{4}{7} \div 5 = \frac{25}{7} \div 5 = \frac{\overset{5}{\cancel{25}}}{7 \times \underset{1}{\cancel{5}}} = \frac{5}{7}. \quad \text{Answer.}$$

Exercise XLIIIa.

[1 *to* 10 *should be taken orally.*]

Divide :—

1. $\frac{4}{5}, \frac{6}{7}, \frac{8}{9}, \frac{10}{17}, \frac{14}{19}, \frac{18}{29}, \frac{32}{33}$ by 2.
2. $\frac{5}{8}, \frac{25}{27}, \frac{35}{64}, \frac{30}{37}, \frac{15}{17}, \frac{10}{13}, \frac{40}{41}$ by 5.
3. $\frac{4}{9}, \frac{3}{5}, \frac{6}{13}, \frac{10}{19}, \frac{15}{17}, \frac{14}{15}$ by 2.
4. $1\frac{2}{7}$ by 3, 4 and 6.
5. $1\frac{5}{19}$ by 3, 5 and 6.
6. $2\frac{1}{22}$ by 5, 3 and 7.
7. $4\frac{5}{7}$ by 5, 9 and 10.
8. $5\frac{4}{55}$ by 3, 4 and 6.
9. $\frac{60}{61}$ by 5, 6 and 15.
10. $\frac{72}{91}$ by 4, 6 and 9.
11. $\frac{55}{56}$ by 5, 11 and 12.
12. $\frac{96}{101}$ by 4, 8 and 12.
13. $\frac{95}{97}$ by 5, 19 and 15.
14. $5\frac{1}{5}$ by 2, 13 and 39.
15. $7\frac{2}{7}$ by 3, 17 and 51.
16. $9\frac{4}{9}$ by 5, 17 and 20.
17. $10\frac{4}{5}$ by 4, 18 and 24.
18. $5\frac{7}{11}$ by 2, 5 and 10.

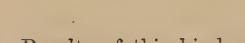
Multiplication by a Fraction

$\frac{5}{6} \times \frac{3}{4}$ means that after taking five-sixths of the unit, we divide the result into 4 equal parts and take three such parts.

Now $\frac{5}{6} \times \frac{1}{4} = \frac{1}{4}$ of $\frac{5}{6} = \frac{5}{6 \times 4}$.

$$\therefore \frac{5}{6} \times \frac{3}{4} = \frac{5}{6 \times 4} \times 3 = \frac{5 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 4} = \frac{5}{8}.$$

Results of this kind may be verified by means of a ruler divided into twelfths, let us say, where AB stands for the unit.



e.g. $\frac{2}{3} \times \frac{1}{4} = \frac{1}{4}$ of $\frac{2}{3} = \frac{1}{4}$ of $AC = AD = \frac{2}{12} = \frac{2 \times 1}{3 \times 4}$.

Ex. $5\frac{2}{5} \times 3\frac{4}{9} = \frac{\overset{3}{\cancel{27}} \times 31}{5 \times \underset{1}{\cancel{9}}} = \frac{93}{5} = 18\frac{3}{5}$. Answer.

Exercise XLIV.

[1 *to* 4 *should be taken orally.*]

Multiply :—

1. $\frac{3}{5}$ by $\frac{5}{7}$, $\frac{10}{21}$, $\frac{1}{2}$.
2. $\frac{4}{7}$ by $\frac{7}{8}$, $\frac{21}{20}$, $\frac{35}{36}$.
3. $1\frac{1}{2}$ by $\frac{4}{9}$, $\frac{4}{27}$, $\frac{5}{18}$.
4. $2\frac{1}{2}$ by $\frac{2}{5}$, $\frac{3}{10}$, $\frac{4}{15}$.
5. $\frac{24}{35}$ by $\frac{10}{12}$ and $\frac{21}{50}$.
6. $\frac{5}{42}$ by $\frac{6}{25}$ and $\frac{28}{35}$.
7. $\frac{9}{140}$ by $\frac{12}{27}$ and $\frac{35}{36}$.
8. $\frac{8}{31}\frac{8}{5}$ by $\frac{63}{64}$ and $\frac{35}{98}$.
9. $\frac{17}{112}$ by $\frac{28}{85}$ and $\frac{24}{119}$.
10. $5\frac{1}{4}$ by $\frac{8}{63}$ and $\frac{9}{17}$.
11. $7\frac{3}{8}$ by $\frac{4}{5}$ and $1\frac{8}{9}$.
12. $12\frac{1}{12}$ by $\frac{6}{25}$ and $2\frac{3}{25}$.
13. $5\frac{11}{13}$ by $\frac{13}{38}$ and $4\frac{1}{19}$.
14. $10\frac{2}{5}$ by $\frac{5}{28}$ and $3\frac{2}{13}$.

Find the value of :—

15. $5\frac{4}{9} \times 1\frac{12}{143} \times 2\frac{1}{7} \times 8\frac{20}{50}$.
16. $\frac{18}{19} \times 182\frac{1}{2} \times \frac{11}{219} \times 4\frac{2}{45}$.
17. $\frac{51}{91} \times 1\frac{27}{68} \times 2\frac{46}{55} \times 1\frac{77}{171}$.
18. $3\frac{1}{18} \times 1\frac{19}{35} \times \frac{7}{11} \times 1\frac{5}{9}$.
19. $2\frac{14}{27} \times 1\frac{11}{13} \times 2\frac{7}{34} \times 1\frac{5}{13}$.
20. $1\frac{15}{22} \times \frac{19}{51} \times 1\frac{76}{111}$.
21. $4\frac{11}{16} \times 1\frac{4}{27} \times 24\frac{12}{85} \times \frac{17}{132}$.
22. $3\frac{9}{16} \times 1\frac{19}{29} \times 1\frac{6}{19} \times 2\frac{17}{35} \times 2\frac{1}{3}$.
23. $11\frac{1}{3} \times 2\frac{5}{14} \times \frac{1}{4} \times 1\frac{9}{11}$.
24. $3\frac{1}{8} \times \frac{4}{5} \times \frac{1}{7} \times 5\frac{1}{4}$.
25. $5\frac{1}{2} \times \frac{5}{27} \times \frac{3}{11} \times \frac{9}{10}$.
26. $4\frac{1}{2} \times 3\frac{2}{3} \times \frac{4}{9} \times \frac{8}{22}$.

Division by a Fraction

To find the value of $\frac{2}{3} \div \frac{3}{5}$.

$$\text{Let } x = \tfrac{2}{3} \div \tfrac{3}{5}.$$
$$\therefore\ x \times \tfrac{3}{5} = \tfrac{2}{3} \div \tfrac{3}{5} \times \tfrac{3}{5} = \tfrac{2}{3}.$$
$$\therefore\ x \times \tfrac{3}{5} \times \tfrac{5}{3} = \tfrac{2}{3} \times \tfrac{5}{3}.$$
$$i.e.\ x = \tfrac{2}{3} \times \tfrac{5}{3}.$$

This result may also be obtained by converting the fractions into new fractions having the same denominator; thus

$$\tfrac{2}{3} \div \tfrac{3}{5} = \tfrac{10}{15} \div \tfrac{9}{15} \text{ (or 10 fifteenths} \div \text{9 fifteenths)}$$
$$= \tfrac{10}{9} = \frac{2 \times 5}{3 \times 3}.$$

Ex. 1. *Divide* $3\frac{1}{21}$ *by* $\frac{4}{7}$.

$$3\tfrac{1}{21} \div \tfrac{4}{7} = \frac{\overset{16}{\cancel{64}}}{\underset{3}{\cancel{21}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{4}}} = \frac{16}{3} = 5\tfrac{1}{3}. \quad \text{Answer.}$$

Ex. 2. *Find the value of* $\dfrac{8\frac{1}{8}}{6\frac{1}{2}}$.

Method i. $\dfrac{8\frac{1}{8}}{6\frac{1}{2}} = 8\tfrac{1}{8} \div 6\tfrac{1}{2} = \dfrac{\overset{5}{\cancel{65}}}{\underset{4}{\cancel{8}}} \times \dfrac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{13}}} = \dfrac{5}{4} = 1\tfrac{1}{4}$. Answer.

Method ii. $\dfrac{8\frac{1}{8}}{6\frac{1}{2}} = \dfrac{8\frac{1}{8} \times 8}{6\frac{1}{2} \times 8} = \dfrac{\overset{5}{\cancel{65}}}{\underset{1}{\cancel{13}} \times 4} = \dfrac{5}{4} = 1\tfrac{1}{4}$. Answer.

Exercise XLIVa.

[1 *to* 4 *should be taken orally.*]

Divide :—

1. $\frac{3}{5}$ by $\frac{6}{7}$, $\frac{11}{10}$, $\frac{3}{10}$, $\frac{6}{11}$.
2. $\frac{2}{7}$ by $\frac{4}{7}$, $\frac{8}{7}$, $\frac{3}{14}$, $\frac{5}{14}$.
3. $\frac{5}{8}$ by $\frac{5}{16}$, $\frac{15}{16}$, $\frac{3}{8}$, $\frac{3}{4}$.
4. $\frac{5}{12}$ by $\frac{5}{8}$, $\frac{10}{12}$, $\frac{11}{16}$, $\frac{3}{8}$.
5. $\frac{15}{17}$ by $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{6}{7}$.
6. $\frac{21}{22}$ by $\frac{3}{5}$, $\frac{9}{11}$ and $\frac{14}{33}$.
7. $1\frac{5}{21}$ by $\frac{2}{5}$, $1\frac{3}{14}$ and $\frac{4}{7}$.
8. $3\frac{3}{5}$ by $\frac{3}{8}$, $1\frac{1}{8}$ and $2\frac{1}{3}$.
9. $5\frac{10}{11}$ by $1\frac{5}{13}$, $4\frac{1}{3}$ and $1\frac{2}{5}$.
10. $\frac{77}{150}$ by $15\frac{31}{30}$.
11. $17\frac{3}{16}$ by $1\frac{23}{256}$.
12. $33\frac{3}{25}$ by $2\frac{91}{265}$.
13. $8\frac{29}{72}$ by $3\frac{61}{108}$.
14. $8\frac{11}{56}$ by $2\frac{13}{70}$.
15. $6\frac{47}{138}$ by $6\frac{21}{184}$.
16. $10\frac{5}{16}$ by $1\frac{13}{64}$.

Find the value of :—

17. $3\frac{1}{11} \div 2\frac{19}{33}$.

18. $23\frac{3}{4} \div 10\frac{25}{36}$.

19. $30\frac{5}{64} \div 21\frac{7}{128}$.

20. $25\frac{5}{46} \div 26\frac{9}{81}$.

21. $\frac{96}{4\frac{16}{29}}$.

22. $\frac{14}{\frac{24}{35}}$.

23. $\frac{2}{1\frac{13}{35}}$.

Easy Miscellaneous Examples on Fractions

Exercise XLV.

Find the value of :—

1. $\frac{9}{77} + 2\frac{7}{33} + \frac{22}{63} + 7\frac{10}{11}$.
2. $120\frac{7}{60} - 9\frac{43}{220}$.
3. $8\frac{1}{7} \div 171$.
4. $6\frac{21}{198} + 5\frac{63}{286} + 19\frac{23}{156} + 27\frac{22}{39}$.
5. $1\frac{67}{186} \times \frac{144}{161} \times 2\frac{55}{112} \times 1\frac{27}{99}$.
6. $19\frac{19}{84} - 14\frac{15}{28}$.
7. $7\frac{7}{44} \times 21\frac{7}{63}$.
8. $7\frac{13}{16} + \frac{17}{72} + 3\frac{2}{9}$.
9. $1\frac{5}{34} \times \frac{85}{252} \times 1\frac{41}{91} \times 2\frac{37}{55}$.
10. $17\frac{21}{68} - 14\frac{49}{51}$.
11. $4\frac{2}{3} + \frac{13}{24}$.
12. $5\frac{7}{24} + 11\frac{19}{21} + 105$.
13. $4\frac{21}{25} \times 2\frac{37}{77}$.
14. $2\frac{5}{8} + 3\frac{3}{8} + 4\frac{1}{15} + 1\frac{17}{20}$.
15. $11\frac{3}{20} - 3\frac{11}{28} - 5\frac{16}{35}$.
16. $21\frac{6}{143} - 20\frac{22}{39}$.
17. $7\frac{46}{51} + 4\frac{95}{136} + 11\frac{11}{357} + 6\frac{83}{168}$.
18. $\frac{1}{2} + \frac{3}{8} + \frac{4}{15} + \frac{7}{24}$.
19. $31\frac{4}{27} \div 122\frac{14}{27}$.
20. $28\frac{17}{143} - 16\frac{9}{13}$.
21. $3\frac{1}{7} + 3\frac{3}{10} + 2\frac{3}{11} + 2\frac{5}{22} + \frac{3}{35}$.
22. $1\frac{11}{45} \times 1\frac{19}{77} \times 1\frac{7}{48}$.
23. $9\frac{9}{10} \div \frac{22}{35}$.
24. $\frac{2}{3} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{9}{10}$.
25. $3\frac{1}{2} \times 2\frac{2}{5} \times \frac{8}{9}$.
26. $13\frac{1}{8} \times 12\frac{3}{5} \times 3\frac{3}{7}$.
27. $220\frac{3}{17} - 14\frac{9}{11}$.
28. $11\frac{19}{25} \times \frac{5}{7} \times \frac{5}{72}$.
29. $\frac{7}{12} \times 3\frac{1}{5} \times 1\frac{1}{14}$.
30. $1\frac{3}{8} + 3\frac{1}{3} - 5\frac{5}{6} + 2\frac{1}{4} - 1\frac{1}{12}$.
31. $9\frac{4}{13} + 14\frac{23}{132} + 3\frac{1}{24} + 7\frac{9}{143}$.
32. $33\frac{3}{7} \div 7\frac{34}{35}$.
33. $3\frac{1}{2} \times 17\frac{2}{5} \times \frac{4}{29} \times 11\frac{1}{63}$.
34. $106\frac{23}{47} \div 17\frac{7}{8}$.
35. $3\frac{1}{5} + \frac{5}{12} + 2\frac{3}{8} + \frac{7}{24}$.

Reduction of one Expression to the Fraction of another

Ex. *Reduce* £2, 10s. 2d. *to the fraction of* 2s. 7½d.

[*i.e.* find the *ratio* of the first quantity to the second.]

£2, 10s. 2d.	2s. 7½d.
20	12
50 s.	31 d.
12	2
602 d.	63 halfpence.
2	
1204 halfpence.	

$$\therefore \frac{\text{£2, 10s. 2d.}}{\text{2s. 7½d.}} = \frac{1204 \text{ halfpence}}{63 \text{ halfpence}} = \frac{1204}{63} = \frac{172}{9} = 19\tfrac{1}{9}. \quad \text{Answer.}$$

Exercise XLVI.

Reduce the first of the following quantities to the fraction of the second :—

1. 2s. 2¼d. and 1 guinea.
2. 7s. 6½d. and £1.
3. £1, 2s. 6d. and £1, 10s. 0d.
4. 18s. 4d. and half-a-guinea.
5. 3s. 4½d. and £1.
6. 16s. and £200.
7. 1 m. 5 dm. 6 cm. and 1 m. 6 dm. 8 cm.
8. 16·5 litres and 18·7 litres.
9. 4 dg. and 1 g. 6 dg. 8 cg.
10. 2 cm. 4 mm. and 8 dm. 8 cm. 8 mm.
11. £3, 2s. 10d. and £5, 10s. 0d.
12. 20 lbs. 15 dwts. and 49 lbs. 10 dwts.
13. 2 lbs. 7 ozs. 5 dwts. and 9 lbs. 10 ozs. 15 dwts.
14. 7 f. 210 yds. and 6 mls. 1 f. 70 yds.
15. 1 qr. 10 lbs. 8 ozs. and 2 qrs. 16 lbs. 3 ozs.
16. 6 cwts. and 16 cwts. 1 qr. 4 lbs.
17. 4 ozs. 10 dwts. 15 grs. and 1 lb. 1 dwt. 16 grs.
18. 16 litres 5 dl. and 1·419 Hl.
19. 3 litres 6 dl. and 9·24 Dl.
20. 6 metres and 10·92 Dm.

UNITARY METHOD

Ex. 1. *If* 15 lbs. *of coffee cost* 22s., *what will be the cost of* 10 lbs. ?

15 lbs. of coffee cost 22s.

$\therefore$ 1 lb. ,, ,, $\frac{22}{15}$ s.

$\therefore$ 10 lbs. ,, ,, $\frac{22 \times 10}{15}$ s.

$\therefore$ Cost required $= \frac{22 \times 2}{3}$ s. $= \frac{44}{3}$ s. $=$ 14s. 8d. Answer.

Ex. 2. *If the carriage of* 2 *tons* 11 *cwts.* 13 *lbs. over a certain distance is* £3, 10*s.* 0*d.*, *what will be the carriage for* 2 *tons* 20 *lbs. over the same distance ?*

The carriage for 5725 lbs. is £$3\frac{1}{2}$

$\therefore$,, ,, ,, 1 lb. ,, £$\frac{3\frac{1}{2}}{5725}$

$\therefore$,, ,, ,, 4500 lbs. ,, £$\frac{3\frac{1}{2} \times 4500}{5725}$

$\therefore$ Cost required $= £\frac{7 \times \overset{180}{\cancel{4500}}}{2 \times \underset{229}{\cancel{5725}}} = £\frac{7 \times 90}{229}$

$= £\frac{630}{229} =$ £2, 15s. 0d. (to the nearest penny).

2 tons 11 cwts. 13 lbs.

20

51 cwts.

112

51

51

115

5725 lbs.

2 tons 20 lbs.

2240

4500 lbs.

Ex. 3. *If* 12 *men do a piece of work in* 10 *days, how long will it take* 40 *men ?*

12 men do the work in 10 days.

$\therefore$ 1 man does ,, ,, 10×12 days.

$\therefore$ 40 men do ,, ,, $\frac{10 \times 12}{40}$ days.

$\therefore$ Time $= \frac{10 \times 12}{40} = 3$ days.

Ex. 4. *Given that* 11·2 *litres of chlorine gas at normal temperature and ressure weigh* 35·5 *grams, find correct*[1] *to two places of decimals the weight f* 73 *litres at the same temperature and pressure.*

11·2 litres of chlorine weigh 35·5 grams.

$\therefore$ 1 litre of chlorine weighs $\frac{35{\cdot}5}{11{\cdot}2}$ gms.

$\therefore$ 73 litres ,, ,, $\frac{35{\cdot}5 \times 73}{11{\cdot}2}$ gms.

$\therefore$ Weight of chlorine $= \frac{35{\cdot}5 \times 73}{11{\cdot}2} = \frac{2591{\cdot}5}{11{\cdot}2}$

$= 231{\cdot}383 \ldots$ gms.

$= 231{\cdot}38$ gms. (correct to 2 places)

$$
\begin{array}{r|l}
 & 231{\cdot}383 \\
1{\cdot}12 & 259{\cdot}15 \\
 & 224 \\
\hline
 & 351 \\
 & 336 \\
\hline
 & 155 \\
 & 112 \\
\hline
 & 430 \\
 & 336 \\
\hline
 & 940 \\
 & 896 \\
\hline
 & 440 \\
 & 336 \\
\hline
\end{array}
$$

Exercise XLVII.

1. If the wages of 12 men for a week is £25, what will be the wages of 4 men for the same time?
2. If 25 men earn £52, 10s. in a given time, how much will 10 men earn at the same rate?
3. Given that 1 oz. of gold is worth £4, 4s., what will be the value of 15 dwts.?
4. If 13 lbs. of sugar can be bought for 2s. 8½d., for how much can 27 lbs. be purchased?
5. If 155 sheep cost £193, 15s., what will be the cost of 172 sheep?
6. Find the income tax on £550 at 1s. 3d. in the £.
7. If 10 articles cost £1, 15s., what will be the cost of 1 gross?
8. The cost of 5 articles is 16s. 0½d.; find the cost of 1 score of similar things.
9. If 20 eggs can be bought for 1s. 3d., how many can be bought for 5s. 3d.?
10. If 70 men do a piece of work in 16 days, how long will it take 28 men to perform the same piece of work?

[1] See note on Contracted Work in Decimals, page 111.

11. If 8 cart-loads of gravel are taken 12 miles for a certain sum of money, how many miles should 5 cart-loads be taken for the same sum of money?
12. If 7 lbs. of apples cost 1s. 5½d., how many pounds can be obtained for 7s. 1d.?
13. The price of 5 oranges is 3d., what should be the price of 80?
14. If the carriage for 2 qrs. 5 lbs. of goods be 10s. 2d., how much will be charged for 3 tons 2 cwts. over the same distance?
15. If the fare for a railway journey of 325 miles be £1, 7s. 1d., what should it be for 631 miles?
16. If a garrison of 45 men have enough food to last them 8 days, then how long would the same food last a garrison of 72 men?
17. The cost of 1 mile 3 fur. of paling is £16, 10s., what should be the cost for 5 miles 5 furlongs?
18. The cost of printing 525 books is £32, 16s. 3d., what will be the cost of printing 1200?
19. It is given that 793·351 cu. inches makes 13 litres; find the number of litres in 1769·783 cu. inches.
20. Find the value of 22920 boxes of matches at 3½d. per dozen.
21. If 7 articles cost £1, 2s. 3¾d., what will be the cost of 1 dozen?
22. The price of 53 eggs is 3s. 3¾d.; how many can be bought for 11s. 6d.?
23. If 7 oranges cost 4d., what will be the cost of 175?
24. If 15 lbs. of apples cost 4s. 4½d., how many pounds can be bought for £1, 7s. 5d.?
25. The cost of fencing a path 55 yards long is 13s. 9d.; what will be the cost of fencing a distance of 1 mile 7 fur. 8 yds.?
26. If 600 books cost £22, 10s. for printing, how many books could be printed for £58, 2s. 6d.?
27. If 13 articles cost £11, 4s. 3d., what will be the price of 1 score?
28. How many pounds of apples can be bought for £1, 2s. 2½d. if 13 pounds cost 3s. 6¼d.?
29. How many oranges can be obtained for £1, 4s. if 11 oranges cost 6d.?
30. The cost of 25 articles is 6s. 9¼d.; what will be the cost of 1 gross?

31. If 27 horses consume a certain amount of fodder in 5 days, in how many days will 35 horses consume the same amount?

32. If 18 lbs. of sugar can be bought for 4s. $10\frac{1}{2}$d., how many pounds could be bought for 14s. $10\frac{3}{4}$d.?

33. If 76 articles cost £25, 0s. 4d., how much will 46 cost?

34. If 2 cwts. 3 qrs. 14 lbs. of sugar cost £6, 14s. 2d., what quantity can be bought for £29, 15s.?

35. If 10 men can mow a field in 12 days, in how many days will 15 men mow it?

36. If a man can do a piece of work in 9 days, working $7\frac{1}{2}$ hours a day, how many hours a day would he have to work in order to do it in $4\frac{1}{2}$ days?

37. How many sovereigns are coined out of 20 lbs. weight of standard gold, which is worth £3, 17s. $10\frac{1}{2}$d. per oz.?

38. If 30 guns in a battle kill 2100 men, how many men would 210 guns kill at the same rate?

39. If £24, 7s. $10\frac{1}{2}$d. be paid as income tax on an income of £650, 10s., what ought to be paid at the same rate on an income of £2450, 6s. 8d.?

40. Find the cost *to the nearest penny* of 509 articles at £1, 6s. 7d. per dozen.

41. If 63 yards of cloth cost £3, 9s. $6\frac{3}{4}$d., what will be the cost of 5 yards?

42. If 13 cwts. of metal cost £16, 14s. $2\frac{1}{2}$d., what will be the cost of 10 cwts. 2 qrs.?

43. The price of 14 sheep and 11 pigs is £51, 10s., and that of 8 sheep and 33 pigs is £69, 10s.; find the value of a sheep and of a pig.

44. An estate pays 4s. $4\frac{1}{2}$d. in the pound; what is the loss on a claim of £865, 4s.?

45. If 8 cwts. 1 qr. 7 lbs. can be bought for £17, 9s. $1\frac{1}{2}$d., what weight can be bought for £43, 1s. $4\frac{1}{2}$d.?

46. If 18 men can do a piece of work in 44 days, in how many days would 26 men do a similar piece of work?

47. The assets of a bankrupt amount to £1251, 6s., and he pays a dividend of 2s. $6\frac{1}{2}$d. in the £; find *to the nearest penny* the amount he owes.

Comparison of Decimals and Vulgar Fractions

Since $\frac{1}{10} = \cdot 1$, $\frac{1}{100} = \cdot 01$, $\frac{1}{1000} = \cdot 001$, etc.

$\therefore \quad \frac{37}{100} = \frac{30}{100} + \frac{7}{100} = \frac{3}{10} + \frac{7}{100} = \cdot 37.$

$2\frac{13}{1000} = 2 + \frac{10}{1000} + \frac{3}{1000} = 2 + \frac{1}{100} + \frac{3}{1000} = 2 \cdot 013.$

Similarly $\cdot 057 = \frac{5}{100} + \frac{7}{1000} = \frac{50+7}{1000} = \frac{57}{1000}.$

***Rule.*—To reduce a decimal fraction to a vulgar fraction, place all the figures after the decimal point in the numerator, omitting any ciphers which come at the beginning, and in the denominator place 1 and as many ciphers as there are figures after the decimal point.**

Ex. 1. *Express* $\frac{281}{125}$ *as a decimal.*

$$\frac{281}{125} = 2\frac{31}{125} = 2\frac{248}{1000} = 2 \cdot 248.$$

Ex. 2. *Express* $\frac{3}{32}$ *as a decimal.*

$$\frac{3}{32} = \cdot 09375.$$

```
4 3
8| ·75
 ·09375
```

Ex. 3. *Express* $1\frac{37}{1600}$ *as a decimal.*

$$1\frac{37}{1600} = \frac{2 \cdot 3125}{100} = \cdot 023125.$$

```
4 37
4| 9·25
 2·3125
```

Ex. 4. *Express* $\frac{5}{27}$ *as a decimal to 5 places.*

$$\frac{5}{27} = \cdot 18518.$$

```
3 5
9| 1·66666 . . . .
 ·18518 . . . .
```

If $\frac{1}{3}$ is converted into a decimal, it becomes $\cdot 333 \ldots\ldots$, the series never terminating; similarly $\frac{2}{11} = \cdot 181818 \ldots\ldots$ These are written $\cdot\dot{3}$ and $\cdot\dot{1}\dot{8}$ respectively. Such decimals, in which either one or more figures continually recur, are called *Recurring Decimals*, dots being placed over the first and last figures of the recurring period; we shall subsequently see how such decimals may be converted into vulgar fractions.

Ex. 5. *Express* $\frac{181}{990}$ *as a recurring decimal.*

$$\frac{181}{990} = \cdot 18\dot{2}.$$

```
9 181
10| 20·11111 . . . .
11|  2·01111 . . . .
   ·18282 . . . .
```

Remember that

$\frac{1}{2} = \cdot5$ $\quad \frac{3}{4} = \cdot75$ $\quad \frac{3}{8} = \cdot375$

$\frac{1}{3} = \cdot\dot{3}$ $\quad \frac{1}{5} = \cdot2$ $\quad \frac{5}{8} = \cdot625$

$\frac{2}{3} = \cdot\dot{6}$ $\quad \frac{1}{8} = \cdot125$ $\quad \frac{7}{8} = \cdot875$

$\frac{1}{4} = \cdot25$

Exercise XLVIII.

[*Examples* 1-6 *should be taken orally.*]

Express as decimals :—

1. $\frac{23}{100}$, $\frac{213}{1000}$, $\frac{207}{1000}$.
2. $\frac{3}{100}$, $\frac{17}{1000}$, $\frac{201}{10000}$.
3. $1\frac{31}{100}$, $2\frac{31}{1000}$, $3\frac{31}{10000}$.
4. $\frac{203}{1000}$, $\frac{2003}{10000}$, $1\frac{2031}{10000}$.
5. $8\frac{211}{1000}$, $6\frac{203}{1000}$, $5\frac{1513}{10000}$.
6. $\frac{71}{1000}$, $\frac{209}{10000}$, $3\frac{20009}{1000000}$.

Express as vulgar fractions in their lowest terms :—

7. ·3, ·03, ·0032.
8. ·412, ·527, 525.
9. ·002, ·0025, ·00625.
10. ·542, ·371, ·00301.
11. ·52, ·502, ·0502.
12. ·77, ·827, ·921.
13. ·01, ·001, ·00101.
14. ·0314, 0021, 5·217.
15. ·395, ·003905, 71·525.
16. 5·075, 50·7055, 85·5255.

Reduce to decimals :—

17. $\frac{9}{20}$, $\frac{4}{25}$, $\frac{7}{50}$, $\frac{11}{50}$.
18. $\frac{5}{8}$, $\frac{7}{160}$, $\frac{11}{64}$, $\frac{1}{32}$.
19. $\frac{69}{625}$, $\frac{103}{125}$, $\frac{113}{250}$, $11\frac{66}{128}$.
20. $\frac{23}{400}$, $\frac{20}{400}$, $\frac{31}{1600}$, $\frac{87}{1600}$, $\frac{142}{1600}$.

Reduce to decimals as far as the 4th place :—

21. $\frac{5}{17}$, $9\frac{13}{42}$, $3\frac{9}{13}$, $3\frac{2}{7}$, $5\frac{1}{27}$.
22. $1\frac{2}{3}$, $\frac{8}{23}$, $3\frac{5}{29}$, $2\frac{4}{51}$, $3\frac{3}{41}$.
23. $\frac{4}{37}$, $3\frac{6}{43}$, $7\frac{9}{47}$, $11\frac{9}{53}$, $5\frac{5}{59}$.

Express as recurring decimals :—

24. $\frac{2}{3}$, $\frac{7}{11}$, $\frac{11}{24}$.
25. $2\frac{22}{37}$, $\frac{11}{101}$, $\frac{5}{28}$.
26. $\frac{12}{35}$, $1\frac{10}{11}$, $1\frac{48}{185}$.
27. $1\frac{39}{41}$, $7\frac{6}{13}$, $\frac{5}{6}$.
28. $\frac{17}{24}$, $5\frac{5}{72}$, $3\frac{12}{45}$.

Miscellaneous Fractions

Rule 1. When two or more fractions are connected by the word 'of' they must be considered as a single term.

Ex. 1. $3\frac{1}{4} \times 1\frac{1}{2} \div 5\frac{1}{2}$ of $7\frac{1}{4} = 3\frac{1}{4} \times 1\frac{1}{2} \div (5\frac{1}{2} \times 7\frac{1}{4})$.

$$= \frac{13}{4} \times \frac{3}{2} \times \frac{2}{11} \times \frac{4}{29} = \frac{39}{319}. \quad \text{Ans.}$$

Rule 2. When fractions are connected by + and − signs, and also × and ÷ signs, then the terms between the + and − must be grouped together as single terms.

Ex. 2. $2\frac{3}{4} + \frac{1}{4} \times \frac{3}{7} \div \frac{1}{8} - \frac{1}{4} = 2\frac{3}{4} + (\frac{1}{4} \times \frac{3}{7} \times \frac{8}{1}) - \frac{1}{4}$.

$$= 2\tfrac{3}{4} + \tfrac{6}{7} - \tfrac{1}{4} = 2\,\frac{21+24-7}{28}$$

$$= 2\tfrac{38}{28} = 3\tfrac{5}{14}. \quad \text{Ans.}$$

Ex. 3. $$\frac{\frac{2}{7} + \frac{3}{11}}{1 - \frac{4}{7} \text{ of } \frac{3}{11}} = \frac{\frac{22+21}{77}}{1 - \frac{12}{77}} = \frac{43}{77} \times \frac{77}{65} = \frac{43}{65}. \quad \text{Ans.}$$

Ex. 4. $$\frac{3}{1 - \frac{1}{3 - \frac{1}{3}}} = \frac{3}{1 - \frac{3}{8}} = \frac{3 \times 8}{5} = 4\tfrac{4}{5}. \quad \text{Ans.}$$

Ex. 5. *Divide* £4, 3s. 2d. *by* 21.

$$\left\{\begin{array}{l|l} 3 & \text{£4, 3s. 2d.} \\ 7 & \text{£1, 7s. } 8\tfrac{2}{3}\text{d.} \\ \hline & \text{3s. } 11\tfrac{1}{21}\text{d.} \quad \text{Ans.} \end{array}\right.$$

Ex. 6. *Find the value of* $3\frac{1}{7}$ of £1, 15s. 7d. + $2\frac{2}{3}$ of £2, 6s. 9d. − $\frac{2}{9}$ of £1, 10s. 9d.

$3\frac{1}{7}$ of £1, 15s. 7d. $= (3 + \frac{1}{7})$ of £1, 15s. 7d. = £5, 6s. 9d.
5s. 1d.

$2\frac{2}{3}$ of £2, 6s. 9d. $= (2 + \frac{2}{3})$ of £2, 6s. 9d. = £4, 13s. 6d. | 3 | £2, 6s. 9d.
£1, 11s. 2d. | 15s. 7d.
£11, 16s. 6d.

$\frac{2}{9}$ of £1, 10s. 9d. = 6s. 10d. | 9 | £1, 10s. 9d.
£11, 9s. 8d. | 3s. 5d.

Exercise XLIX.

Simplify :—

1. $\dfrac{\frac{2}{3}+\frac{4}{5}\text{ of }\frac{5}{9}-\frac{8}{21}}{1+\frac{2}{3}\times\frac{3}{7}-\frac{5}{9}}$.

2. $\dfrac{4\frac{1}{2}-3\frac{1}{8}+5\frac{1}{12}}{7\frac{1}{2}-4\frac{1}{3}+11\frac{1}{12}}-\dfrac{11\frac{3}{4}-5\frac{7}{15}}{11\frac{3}{4}+5\frac{7}{15}}$.

3. $\dfrac{3\frac{1}{2}+2\frac{1}{4}}{3\frac{1}{2}-2\frac{1}{4}}$.

4. $\dfrac{\frac{27}{64}-\frac{8}{27}}{\frac{9}{16}+\frac{1}{2}+\frac{4}{9}}$.

5. $\dfrac{2\frac{1}{4}-1\frac{1}{3}}{3\frac{1}{8}+1\frac{1}{4}}$.

6. $\dfrac{2\frac{1}{2}}{54}+\frac{1}{8}-\dfrac{1\frac{1}{16}+\frac{1}{12}}{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}$.

7. $\dfrac{\frac{13}{42}\text{ of }\frac{1}{6}-\frac{11}{28}\text{ of }\frac{1}{9}}{\frac{16}{42}\text{ of }\frac{1}{6}-\frac{13}{28}\text{ of }\frac{1}{9}}$.

8. $\dfrac{15\frac{3}{4}-1\frac{4}{3}\times 1\frac{5}{6}}{\frac{1}{5}\times 23\frac{1}{3}+9\frac{1}{36}}$.

9. $\dfrac{7\frac{2}{5}\times 7\frac{2}{5}\times 7\frac{2}{5}-27}{7\frac{2}{5}\times 7\frac{2}{5}-9}$.

10. $\dfrac{2\frac{1}{3}-\frac{7}{9}+\frac{2}{7}}{1+\frac{6}{7}-\frac{5}{6}}$.

11. $\dfrac{\frac{9}{7}-\frac{2}{3}+\frac{1}{5}}{\frac{6}{7}+\frac{3}{5}-\frac{1}{6}}$.

12. $10\frac{5}{6}\times 1\frac{1}{13}\frac{2}{3}-1\frac{9}{35}+\frac{3}{7}\text{ of }3\frac{3}{4}$.

13. $\dfrac{3\frac{3}{4}+2\frac{5}{8}-4\frac{1}{16}}{3\frac{1}{12}\times 2\frac{2}{5}\div 2\frac{1}{2}}$.

14. $5-[7\frac{1}{2}-(3\frac{1}{3}+2\frac{1}{4})]+2\frac{1}{3}$.

15. $8\frac{1}{3}-[4\frac{1}{3}-\{2\frac{1}{2}-3\frac{1}{3}+(5\frac{1}{2}-2\frac{1}{4})\}]$.

16. $5\frac{1}{10}-7\frac{1}{2}+\{2\frac{1}{3}-3\frac{1}{4}-(1\frac{1}{4}-5\frac{2}{3})+5\}$.

17. $23\frac{7}{10}-5\frac{1}{5}-[2\frac{1}{3}-5\frac{1}{7}+(2\frac{1}{2}-3)-3]$.

18. $5-\dfrac{1}{3-\frac{1}{3}}$.

19. $\dfrac{6}{4-\dfrac{3}{2-\frac{1}{5}}}$.

20. $\dfrac{5}{4-\dfrac{1}{9+\frac{2}{7}}}$.

21. $2-\dfrac{3}{3-\dfrac{2\frac{1}{2}+\frac{1}{3}}{3-\frac{1}{4}}}$.

22. $3-\dfrac{3}{2-\frac{3}{4}}$.

23. $5-\dfrac{2}{4-\dfrac{1}{1-\frac{1}{2}}}$.

Divide :—

24. £7, 4s. 11d. by 21 by short division.
25. £4, 5s. 9d. by 24 „ „
26. £11, 3s. 1d. by 28 „ „
27. £4, 9s. 8d. by 55 „ „
28. £7, 5s. 2d. by 33 „ „
29. £8, 2s. $2\frac{1}{2}$d. by 35 „ „
30. What fraction of 35s. is equal to the sum of $2\frac{3}{4}$ of 7s. 6d. and $\frac{3}{2}\times\frac{5}{4}$ of 3s. ?
31. Find the value of $\frac{3}{4}$ of £5, 7s. 11d. + $\frac{4}{5}$ of £20, 5s. $9\frac{3}{4}$d. − $\frac{16}{9}$ of £11, 12s. 6d.

32. What fraction of £24 is $\frac{3}{7}$ of £1, 10s. ?

33. Find the value of $\frac{3}{10}$ of £2 + $\frac{4}{7}$ of half-a-guinea + 8s. 8d.

34. What fraction of a crown is the difference between $\frac{3}{4}$ of a shilling and $\frac{5}{8}$ of a guinea ?

35. Find the value of $\frac{5}{6}$ of £75, 16s. 8d. + $\frac{1}{3}$ of £2, 10s. 2d., and determine what fraction £10, 2s. 6d. is of £151, 17s. 6d.

36. Find the value of—
$7\frac{1}{4}$ of £3, 13s. $8\frac{1}{2}$d. + $2\frac{1}{10}$ of £5, 16s. $9\frac{1}{4}$d. − $3\frac{1}{3}$ of £1, 17s.

Miscellaneous Problems on Vulgar Fractions

Exercise L.

1. What must be added to $13\frac{4}{9}$ that the result may be $15\frac{7}{8}$?

2. Find the excess of $17\frac{2}{5}$ over $14\frac{2}{9}$.

3. By how much does the sum of $3\frac{1}{7}$, $5\frac{2}{3}$, $8\frac{1}{7}$ exceed the difference between $8\frac{2}{5}$ and $4\frac{1}{3}$?

4. Find the excess of the product of $5\frac{2}{5}$ and $4\frac{1}{3}$ over $4\frac{1}{4}$.

5. Multiply the product of $7\frac{1}{5}$ and $2\frac{5}{6}$ by the quotient of $4\frac{1}{3}$ by $1\frac{2}{9}$.

6. Multiply the difference between $15\frac{2}{7}$ and $4\frac{1}{3}$ by the difference between $14\frac{2}{5}$ and $4\frac{1}{5}$.

7. Divide the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ by the sum of $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

8. Divide the difference between $3\frac{1}{7}$ and $2\frac{1}{5}$ by the sum of $\frac{2}{7}$ and $4\frac{1}{5}$.

9. A man travels $\frac{3}{8}$ of his journey by rail, $\frac{4}{9}$ by motor car, and walks 13,000 yards. Find the length of the total journey.

10. From a piece of timber 29 feet in length, as many pieces as possible $3\frac{2}{3}$ feet in length are cut. What is the length of the remaining piece ?

11. One-seventh of a number added to one-fifth of the same number makes 144 ; find the number.

12. A and B receive respectively $\frac{3}{13}$ and $\frac{5}{17}$ of a certain legacy, and C the remainder. If A receives £5100, how much do B and C receive?

13. If $\frac{2}{3}$ of an estate is worth £13,768, what is the value of $\frac{3}{4}$ of it?

14. If I pay away $\frac{1}{4}$ of my money, then $\frac{1}{5}$ of what remains, and finally $\frac{1}{6}$ of what still remains, what fraction of the original money is left?

15. One-third of the beer from a cask runs away owing to leakage; 4 gallons are drawn off, and the cask is then found to be half full; how many gallons will the cask hold?

16. One-third of an estate belongs to A, two-fifths to B, and the remainder to C. A's share is worth £1000 more than C's; what is the value of the whole estate?

17. If in a certain transaction a man loses $\frac{2}{3}$ of his goods and then has £2465 left, what was the original value of his goods?

18. From a pole 37 feet long, as many pieces as possible of $2\frac{1}{5}$ feet are cut; how much of the pole is left?

19. Three people own a piece of ground worth £2450; one has $\frac{3}{5}$ and the second $\frac{2}{7}$ of it; what is the value of the third person's share?

20. One person having £132 has $3\frac{1}{5}$ times as much as a second person; how much has the second person?

21. A man leaves a sum of money to be divided into 13 shares, of which one daughter receives 3, and another 5, and the third the rest, which amounts to £1375. What is the value of the sum of money?

22. How many pieces of string, each $\frac{3}{4}$ foot in length, can be cut from a piece $5\frac{1}{2}$ yards long?

23. In a certain park, one-third of the trees are oaks, one-quarter elms, and one-fifth beeches, and the remainder are 169 in number. How many trees are there altogether?

24. A person spends $\frac{5}{17}$ of his money and then $\frac{1}{3}$ of the remainder, and finds he has £824 left. How much had he at first?

Decimalisation of Money and Reduction of one quantity to the Decimal of another

Learn :—

$$2s. = £\cdot 1$$
$$1s. = £\cdot 05$$
$$6d. = £\cdot 025$$
$$\tfrac{1}{4}d. = £\cdot 001\tfrac{1}{24}.$$

By means of these values, any sum of money may easily be reduced to the decimal of £1.

Ex. 1. *Reduce* 3s. 6d. *to the decimal of* £1.

$$\begin{aligned} 3s.\ 6d. = 2s. + 1s. + 6d. &= £\cdot 1 \\ &\quad \cdot 05 \\ &\quad \cdot 025 \\ &= £\cdot \overline{175} \end{aligned}$$

Ex. 2. *Reduce* £3, 17s. $4\tfrac{3}{4}$d. *to the decimal of* £5.

$$\begin{aligned} £3,\ 17s.\ 4\tfrac{3}{4}d. &= £3 + 16s. + 1F. + 19 \text{ farthings} \\ &= £3 + \cdot 8 + \cdot 05 + \cdot 019\tfrac{19}{24} \\ &= 3\cdot 85 \\ &\quad \cdot 019791\dot{6} \\ &= £3\cdot 869791\dot{6} \end{aligned}$$

$$\begin{array}{r|l} 4 & 19 \\ \hline 6 & \underline{4\cdot 75} \\ & \cdot 791\dot{6} \end{array}$$

$$\therefore \frac{£3,\ 17s.\ 4\tfrac{3}{4}d.}{£5} = \cdot 773958\dot{3}.$$

Alternative Method :—

$$\begin{array}{r|l} 12 & 4\cdot 75\ d. \\ \hline 20 & \underline{17\cdot 39583\dot{3}\ s.} \\ 5 & \underline{3\cdot 869791\dot{6}\ £} \\ & \cdot 773958\dot{3} \end{array}$$

Ex. 3. *Reduce* 1 lb. 5 ozs. *to the decimal of* 1 cwt.

$$\left\{\begin{array}{r|l} 4 & 5\ ozs. \\ \hline 4 & \underline{1\cdot 25} \end{array}\right.$$

$$\left\{\begin{array}{r|l} 4 & 1\cdot 3125\ lb. \\ \hline 7 & \underline{\cdot 328125} \\ 4 & \underline{\cdot 046875\ qr.} \\ & \cdot 01171875\ cwt. \end{array}\right.$$

If the second expression is a compound quantity, the following method should be adopted :—

Ex. 4. *Reduce* 16s. $1\frac{1}{2}$d. *to the decimal of* £5, 7s. 6d.

16s. $1\frac{1}{2}$d.	£5, 7s. 6d.
12	20
193 d.	107 s.
2	12
387 halfpence.	1290 d.
	2
	2580 halfpence.

$\therefore$ Decimal $= \frac{387}{2580} = \frac{3}{20} = \cdot 15$.

Approximate Decimalisation of Money

If the sum of money is required correct to 3 decimal places, then, since 12 farthings $= £\cdot 012\frac{1}{2}\frac{2}{4} = £\cdot 0125 = £\cdot 013$ (correct to 3 places), we may *substitute for each farthing,* £·001, *adding* 1 *to the third decimal place if the number of farthings is between* 12 *and* 23 *inclusive.*

Ex. 5. *Find the value of* £5, 7s. $2\frac{1}{4}$d. *as the decimal of* £1 *correct to* 3 *places.*

£5, 7s. $2\frac{1}{4}$d. = £5 + 6s. + 1s. + 9 farthings.
= £5
·3
·05
·009
= £5·359.

Ex. 6. *Find the value of* £3, 15s. $10\frac{1}{2}$d. *as the decimal of* £1 *correct to* 3 *places.*

£3, 15s. $10\frac{1}{2}$d. = £3 + 14s. + 1s. + 6d. + 18 farthings.
= £3
·7
·05
·025
·019
= £3·794.

Exercise LI.

[*Examples* 1 *to* 6 *should be taken orally.*]

Reduce to the decimal of £1 :—

1. 6s., 10s., 16s., 18s.
2. 7s., 11s., 17s., 19s.
3. 2s. 6d., 5s., 7s. 6d., 12s. 6d., 17s. 6d., 15s.
4. 5s. 6d., 8s. 6d., 10s. 6d., 18s. 6d., 16s. 6d., 11s. 6d.
5. £1, 10s. 6d., £2, 5s. 0d., £1, 8s. 6d., £3, 6s. 6d., £5, 14s. 6d.
6. $4\frac{3}{4}$d., $5\frac{1}{2}$d., $2\frac{1}{2}$d., $1\frac{3}{4}$d., $5\frac{1}{4}$d., $3\frac{1}{2}$d. (correct to 3 places).
7. 2s. 6d., 4s. 6d., 17s. 6d., 11s. 3d.
8. 13s. $1\frac{1}{2}$d., 17s. $4\frac{1}{2}$d., 15s. $3\frac{3}{4}$d., 3s. $4\frac{1}{4}$d.
9. £3, 7s. $2\frac{1}{4}$d., £2, 9s. $5\frac{1}{4}$d., £5, 10s. $3\frac{3}{4}$d.
10. £12, 5s. $7\frac{1}{2}$d., £5, 11s. 3d., £9, 17s. $4\frac{1}{2}$d.
11. £9, 14s. $9\frac{3}{4}$d., £21, 19s. $11\frac{1}{4}$d., £42, 13s. $4\frac{1}{4}$d.
12. £1, 15s. $4\frac{1}{4}$d., £2, 13s. $2\frac{1}{2}$d., £7, 9s. $11\frac{1}{4}$d.

Reduce :—

13. £4, 15s. $7\frac{1}{2}$d. to the decimal of £5.
14. £10, 11s. $9\frac{3}{4}$d. „ „ £10.
15. 6s. $6\frac{3}{4}$d. „ „ 5s. 3d.
16. £3, 12s. $2\frac{1}{4}$d. „ „ £3, 17s.
17. £1, 2s. $3\frac{1}{4}$d. „ „ £17, 16s. 4d.
18. £36, 17s. $8\frac{3}{4}$d. „ „ £5.
19. £1, 2s. 6d. „ „ £1, 17s. 6d.
20. 2 cwts. 3 qrs. 3 lbs. 8 ozs. to the decimal of 1 ton.
21. 1 oz. 5 grs. to the decimal of 3 ozs. 4 dwts. 16 grs.
22. 2 cwts. 1 qr. 27 lbs. to the decimal of 10 cwts. 5 lbs.
23. 5 lbs. 10 ozs. 11 dwts. to the decimal of 14 lbs. 2 ozs.
24. 1 ton 19 cwts. 7 lbs. 2 ozs. to the decimal of 9 tons 15 cwts. 1 qr. 7 lbs. 10 ozs.
25. 2 cwts. 2 qrs. 8 lbs. to the decimal of 6 cwts. 3 qrs. 12 lbs.
26. 2 fur. 41 yds. to the decimal of 1 mile.
27. 3 fur. 33 yds. to the decimal of 1 mile.

Find the value as the decimal of £1, *correct* to three places, of each of the following expressions :—

28. £3, 2s. $4\frac{1}{4}$d.
29. £5, 10s. 6d.
30. £4, 13s. $6\frac{1}{2}$d.
31. £10, 15s. $10\frac{3}{4}$d.
32. £8, 9s. $11\frac{1}{2}$d.
33. £4, 3s. $4\frac{1}{4}$d.
34. £10, 15s. $7\frac{3}{4}$d.
35. £15, 7s. $9\frac{1}{2}$d.
36. £20, 19s. $10\frac{1}{4}$d.
37. £17, 3s. $0\frac{1}{4}$d.
38. £22, 13s. $5\frac{1}{2}$d.
39. £25, 9s. $4\frac{1}{2}$d.
40. £27, 10s. $11\frac{3}{4}$d.

To find the value of the Decimal of a given Quantity

Ex. 1. *Find the value of* ·675 *of* £3.

$$\text{Exp.} = \tfrac{675}{1000} \text{ of } £3 = £\tfrac{27}{40} \times 3 = £\tfrac{81}{40} = £2,\ 0\text{s. } 6\text{d.}$$

Ex. 2. *Evaluate* ·637 *of* £5.

·637
5
3·185 £
20
3·7 s.
12
8·4 d.

∴ Exp. = £3, 3s. 8·4d.

Ex. 3. *Find the value of* ·356 *of* £2, 17s. $1\frac{1}{2}$d.

£2, 17s. $1\frac{1}{2}$d.
20
57 s.
12
685·5d.

68·55
3·56
205·65
34·275
4·1130
244·038 d. = 20s. 4·038d.

∴ Exp. = ·356 × 685·5d. = £1, 0s. 4·038d.

Approximate Methods

If the value of a sum of money expressed as a decimal of £1 is to be found *correct to the nearest penny*, it should firstly be written down *correct to three places of decimals.*

1. The first figure after the decimal point gives the number of florins.

2. Dividing the number obtained from the second and third figures by 25, the quotient gives the number of sixpences and the remainder the number of farthings ; (1 should, however, be subtracted from the remainder if the number of farthings so obtained is greater than 12).

[This method will in *nearly* all cases give the answer correct to the nearest farthing also.]

Ex. 1. *Find correct to the nearest penny, the value of* £0·5672.

£0·567 = 5 florins + 2 sixpences + (17 − 1) farthings,
= 11s. 4d.

Ex. 2. *Find correct to the nearest penny, the value of* £5·53373.

£5·53373 = £5·534 (approx.) = £5 + 5 florins + 1 sixpence + 9 farthings,
= £5, 10s. 8d. (correct to nearest penny).

Alternative Method :—

£5·53373
20
10·6746
12
8·0952

∴ Exp. = £5, 10s. 8d.

There are a few cases (in which the true value of the sum of money ends with *nearly* a halfpenny) in which the method given in Ex. 1 fails ; it is then advisable to retain four figures beyond the decimal point, *e.g.* :—

£·0187 = ·374s. = 4·488d. = 4d. (to the nearest penny) ;

by the approximate method, £·0187 = £·019 = 18 farthings = $4\frac{1}{2}$d.
= 5d. (correct to the nearest penny).

In such a case the following method may be used :—

Ex. 3. £·0187 = (18·7 − 1) farthings.

= 17·7 farthings = 4¼d. + $\frac{7}{10}$ of a farthing.

= 4d. (to the nearest penny).

Exercise LII.

Find the value of :—

1. 5·25 of £4.
2. 4·1325 of 5s.
3. 12·45 of £9.
4. 3·145 of 15s.
5. ·925 of 12s. 6d.
6. 3·255 of £1, 10s.
7. ·536 of £1, 17s. 6d.
8. ·0828125 of 1 ton.
9. ·0053125 of £20.
10. 2·75 of £1, 7s. 6d.
11. ·378125 of £1 + ·8375 of 5s. + ·0375 of 6s. 8d.
12. ·8752 of 1 guinea − ·52521 of £1.

Find, *correct to the nearest penny*, the value of :—

13. £0·7324.
14. £0·7829.
15. £0·94213.
16. £0·704.
17. £0·8234.
18. £0·5206.
19. £0·4123.
20. £0·49072.
21. £0·6315.
22. £0·42871.
23. £0·50203.
24. £0·30907.
25. ·056 of £2, 10s. 6d. + 3$\frac{1}{5}$ of £3, 7s. 7d.
26. ·045 of £7, 10s. 6d. + 3$\frac{2}{5}$ of £8, 1s. 7d.
27. ·072 of £2, 10s. 5d. + 1$\frac{7}{9}$ of £8, 2s. 6d.
28. Find the value of ·01625 of £204, 3s. 4d.

SIMPLE PRACTICE

[The working should, if necessary, be carried on for 2 decimal places.]

Ex. 1. *Find the value of* $23\frac{1}{4}$ *lbs. at* 8s. 4d. *a lb.*

Cost at £1 per lb.	£23, 5s. 0d.
5s. $= \frac{1}{4}$ of £1.	£5, 16s. 3d.
3s. 4d. $= \frac{1}{6}$ of £1.	3, 17s. 6d.
	£9, 13s. 9d.

Ex. 2. *Find the value of* $523\frac{3}{7}$ *articles at* 17s. 3d. *each.*

Cost of 523 articles at £1 each.	£523 0 0
10s. $= \frac{1}{2}$ of £1	£261 10 0
5s. $= \frac{1}{2}$ of 10s.	130 15 0
2s. $= \frac{1}{5}$ of 10s.	52 6 0
3d. $= \frac{1}{8}$ of 2s.	6 10 9
$\frac{3}{7}$ article at 17s. 3d.	0 4 11·14
	£451 6 8·14

$$7\overline{)34\ 6}\quad 4\ 11{\cdot}14$$

Exercise LIII.

Find by Practice (working the pence to 2 decimal places) the value of :—

1.	25	articles at	£1	15	0	each.
2.	32	,, ,,	2	10	0	,,
3.	34	,, ,,	2	10	6	,,
4.	52	,, ,,	5	11	6	,,
5.	39	,, ,,	2	6	8	,,
6.	42	,, ,,	4	13	4	,,
7.	57	,, ,,	5	7	6	,,
8.	95	,, ,,	10	17	6	,,
9.	135	,, ,,	5	13	$4\frac{1}{2}$	,,
10.	152	,, ,,	7	14	$4\frac{1}{2}$	,,
11.	144	,, ,,	28	13	$7\frac{1}{2}$	,,
12.	82	,, ,,	0	6	$5\frac{1}{2}$	,,
13.	1842	,, ,,	2	6	$9\frac{1}{4}$	,,
14.	132	,, ,,	17	1	$4\frac{1}{2}$	,,

15.	121	articles at	£8	16	8	each.
16.	113	„ „	4	8	6	„
17.	45	„ „	16	8	$1\frac{1}{4}$	„
18.	65	„ „	4	18	9	„
19.	490	„ „	2	14	$11\frac{1}{4}$	„
20.	79	„ „	24	0	$10\frac{1}{2}$	„
21.	465	„ „	31	14	$9\frac{1}{2}$	„
22.	495	„ „	24	3	$1\frac{1}{2}$	„
23.	249	„ „	11	17	$9\frac{1}{2}$	„
24.	651	„ „	651	1	$2\frac{1}{2}$	„
25.	856	„ „	27	14	$3\frac{1}{2}$	„
26.	$3\frac{33}{80}$	„ „	4	16	8	„
27.	$1435\frac{1}{4}$	„ „	0	14	3	„
28.	$517\frac{2}{3}$	„ „	3	10	$1\frac{1}{2}$	„
29.	$255\frac{2}{5}$	„ „	4	12	$4\frac{1}{2}$	„
30.	$14\frac{3}{16}$	„ „	2	18	4	„

COMPOUND PRACTICE

Ex. 1. *Find, correct to the nearest penny, the value of* 10 ozs. 13 dwts. *at* £4, 15s. 4d. *per* oz.

Cost of 1 oz.	£4	15	4
			10
	47	13	4
10 dwts. $= \frac{1}{2}$ of 1 oz.	2	7	8
2 dwts. $= \frac{1}{5}$ of 10 dwts.		9	6·4
1 dwt. $= \frac{1}{2}$ of 2 dwts.		4	9·2
	£50	15	4

Ex. 2. *Find, correct to the nearest penny, the cost of* 5 cwts. 3 qrs. 5 lbs. *at* £1, 17s. 5d. *per* qr.

Cost of 1 qr.	£1	17	5
			23
	43	0	7
4 lbs. $= \frac{1}{7}$ of 1 qr.		5	4·14
1 lb. $= \frac{1}{4}$ of 4 lbs.		1	4·03
	£43	7	3

Exercise LIV.

Find by Practice, *correct to the nearest penny*, the value of :—

1. 3 lbs. 4 ozs. 10 dwts. at £4, 2s. per lb.
2. 12 lbs. 5 ozs. 15 dwts. at £3, 4s. 6d. per lb.
3. 9 lbs. 10 ozs. 16 dwts. at £5, 2s. 6d. per lb.
4. 10 lbs. 6 ozs. 12 dwts. at £3, 2s. 8d. per oz.
5. 12 lbs. 7 ozs. 13 dwts. 12 grs. at £5, 2s. 1d. per oz.
6. 4 lbs. 5 ozs. 5 dwts. 21 grs. at £4, 3s. 4d. per oz.
7. 13 lbs. 8 ozs. 14 dwts. at £3, 4s. 6d. per lb.
8. 6 tons 12 cwts. 3 qrs. 27 lbs. at £62, 17s. 8d. per cwt.
9. 2 tons 3 cwts. 2 qrs. 21 lbs. at £4, 10s. per cwt.
10. 22 cwts. 3 qrs. 21 lbs. at £12, 16s. 8d. per cwt.
11. 12 tons 2 cwts. 3 qrs. 11 lbs. at £17, 11s. 8d. per ton.
12. 3 tons 5 cwts. 27 lbs. 13 ozs. at £4, 13s. 4d. per qr.
13. 114 tons 3 cwts. 15 lbs. 8 ozs. at £422, 6s. 8d. per ton.
14. 39 cwts. 3 qrs. 26 lbs. at £2, 8s. 5d. per cwt.
15. 3 tons 11 cwts. 3 qrs. 10 lbs. 8 ozs. at £4, 3s. 4d. per ton.
16. 10 tons 13 cwts. 2 qrs. 18 lbs. at £1, 3s. 4d. per ton.
17. 42 tons 1 cwt. 1 qr. at £43, 7s. 2½d. per cwt.
18. 27 tons 14 cwts. 3 qrs. 14 lbs. at £4, 16s. per ton.
19. [illegible]iles 4 fur. 165 yds. at £427, 13s. 4d. per mile.
20. 5 [illegible]iles 6 fur. 175 yds. at £320, 5s. 6d. per mile.
21. [illegible]3 acres 3 rds 15 perches at £9, 17s. 4d. per acre.
22. 15 acres 3 rds 17 perches at £15, 13s. 4d. per acre.
23. 10 acres 3 rds. 26 perches at £2, 8s. per acre.

Find by Practice, *correct to the nearest penny*, the dividend on :—

24. £387, 12s. 6d. at 12s. 6d. in the £.
25. £4275, 5s. 6d. „ 13s. 2d. „ „
26. £3256, 7s. 0d. „ 14s. 6d. „ „
27. £5281, 15s. 6d. „ 5s. 9d. „ „
28. £4296, 10s. 0d. „ 11s. 3d. „ „

Areas and Volumes—*continued*

Ex. 1. *Find the number of rectangular tiles,* 1 ft. *by* 8 ins., *required to cover the floor of a hall* 40 ft. *long and* 15 ft. *broad.*

Area of hall $= 40 \times 15$ sq. ft.
Area of 1 tile $= 1 \times \frac{2}{3}$ sq. ft. $= \frac{2}{3}$ sq. ft.

$$\therefore \text{ No. of tiles} = \frac{40 \times 15}{\frac{2}{3}} = \frac{\overset{20}{\cancel{40}} \times 15 \times 3}{\underset{1}{\cancel{2}}} = 900.$$

Ex. 2. *Find the number of sq. feet to be painted inside an open cistern whose internal dimensions are* 4 ft. *long,* 3 ft. 6 ins. *broad, and* 3 ft. 8 ins. *deep.*

Total area = area of 4 sides + area of bottom
$= (\text{perimeter} \times \text{height}) + (\text{length} \times \text{breadth})$
$= (15 \times 3\frac{2}{3}) + (4 \times 3\frac{1}{2})$ sq. ft.
$= \frac{15 \times 11}{3} + \frac{4 \times 7}{2}$ sq. ft.
$= 55 + 14 = 69$ sq. ft.

Ex. 3. *What area of planking* ·5 centimetre *thick will be required for a cubical box, the external edge of which is* 1·51 metres?

Volume of wood required $= (1{\cdot}51)^3 - (1{\cdot}5)^3$ cu. m.
$= 3{\cdot}442951 - 3{\cdot}375$ cu. m.
$= {\cdot}067951$ cu. m.

$\therefore$ Area of planking $= {\cdot}067951 \div {\cdot}005$ sq. m.
$= 13{\cdot}5902$ sq. m.

1·51
1·51
1·51
·75
·01
2·280
1·51
2·2801
1·14005
·022801

1·5
1·5
1·5
·75
2·25
1·5
2·25
1·125

Ex. 4. *If a map is drawn to a scale of* $\frac{1}{2000}$, *what is the real area of a field in acres, which on the map is* 4 inches *long and* $3\frac{1}{2}$ inches *broad?*

$$\text{True length} = 4 \times 2000 \text{ ins.}$$

$$\text{True breadth} = \frac{7 \times 2000}{2} \text{ ins.}$$

$$\therefore \text{Area} = \frac{\overset{500}{\cancel{8000}} \times \overset{175}{\cancel{7000}}}{\underset{9}{\cancel{144}} \times 9 \times \underset{121}{\cancel{14520}}} \text{ acres} = \frac{87500}{9801} = 8\frac{9092}{9801} \text{ acres.}$$

Ex. 5. *Find the cost of papering the walls of a room* 18 ft. 4 ins. *long,* 17 ft. 8 ins. *broad, and* 12 ft. *high, with paper* 3 ft. *wide at* $2\frac{1}{4}$d. *per* yard.

$$\text{Area of walls} = \text{perimeter} \times \text{height} = 72 \times 12 \text{ sq. ft.}$$

$$\therefore \text{Length of paper} = \frac{\text{area}}{\text{width}} = \frac{72 \times 12}{3} \text{ ft.} = 288 \text{ ft.} = 96 \text{ yds.}$$

$$\therefore \text{Cost of paper} = 2\tfrac{1}{4}\text{d.} \times 96 = 9 \times 24\text{d.} = 18\text{s.}$$

N.B.—It may sometimes be useful to use the formula

$$x^2 - y^2 = (x+y)(x-y)$$

e.g. $545^2 - 543^2 = 1088 \times 2 = 2176.$

Exercise LV.

Find in sq. feet the **areas** of each of the following rectangles :—

1. Length = 12 ft. 3 ins., breadth = 7 ft. 2 ins.
2. Length = 5 ft., breadth = 3 ft. 1 in.
3. Length = 13 ft. 5 ins., breadth = 10 ft. 4 ins.
4. Length = 10 ft. 9 ins., breadth = 5 ft. 7 ins.
5. Length = 12 ft. 3 ins., breadth = 8 ft.
6. Length = 31 ft. 7 ins., breadth = 21 ft.
7. Length = 32 ft. 3 ins., breadth = 23 ft. 1 in.

Find the breadth of the following rectangles, the areas and lengths being given :—

8. Area = 433 sq. yds. 3 sq. ft., length = 97 ft. 6 ins.
9. Area = 15 sq. yds. 5 sq. ft., length = 4 yds. 2 ft.
10. Area = 25 sq. yds., length = 7 yds. 1 ft. 6 ins.
11. Area = 25 sq. yds. 4 sq. ft. 72 sq. ins., length = 17 yds.
12. Area = 157 sq. ft. 84 sq. ins., length = 15 ft. 6 ins.

Find in sq. feet and sq. inches the areas of the four walls of the following ooms :—

3. Length = 20 ft., breadth = 15 ft., height = 10 ft. 6 ins.
4. Length = 17 ft. 6 ins., breadth = 14 ft., height = 9 ft. 4 ins.
5. Length = 15 ft. 4 ins., breadth = 12 ft. 3 ins., height = 8 ft.
6. Length = 18 ft. 3 ins., breadth = 15 ft. 6 ins., height = 9 ft. 6 ins.
7. Length = 17 ft. 6 ins., breadth = 14 ft. 2 ins., height = 8 ft. 6 ins.
8. Length = 21 ft. 3 ins., breadth = 19 ft. 6 ins., height = 11 ft.

Find the **volume** in cubic feet of the following rectangular solids :—

9. Length = 18 ft., breadth = 15 ft., height = 10 ft.
0. Length = 17 ft. 6 ins., breadth = 15 ft. 4 ins., height = 12 ft.
1. Length = 16 ft. 3 ins., breadth = 14 ft. 2 ins., height = 10 ft. 6 ins.
2. Length = 25 ft., breadth = 22 ft. 6 ins., height = 25 ft. 4 ins.
3. Length = 25 ft. 6 ins., breadth = 23 ft. 4 ins., height = 12 ft. 6 ins.
4. Length = 15 ft. 6 ins., breadth = 12 ft. 4 ins., height = 7 ft. 3 ins.
5. A rectangular piece of ground, 100 ft. by 72 ft., has a grass plot in the centre and a path 4 ft. wide all round ; what is the area in sq. ft. of the path ?
6. A garden is 50 ft. 4 ins. long and 30 ft. 5 ins. wide, and has a gravel path 3 ft. wide all round ; express the area of the path in sq. ft.
7. If the grass plot of a garden measures 53 ft. by 43 ft. and there is a path 3 ft. wide all round, what is the total area of the garden in sq. ft. ?
8. If the grass plot of a garden is a square whose side is 54 ft. and there is a path 4 ft. wide all round, what will be the area of the path in sq. yds. and sq. ft. ?
9. What will be the cost of carpeting a room 16 ft. 6 ins. long and 10 ft. 3 ins. broad at 4s. per sq. yd. ?
0. The cost of carpeting a room is £6, 10s. $6\frac{1}{3}$d. ; if the width of the room is 18 ft. 6 ins. and the carpet costs 3s. per sq. yd., what is the length of the room ?
1. How many tiles each 1 ft. long and 6 ins. broad will be required to cover a hall which measures 20 ft. 6 ins. by 7 ft. ?

32. How many blocks of wood 6 ins. square will be required to cove the floor of a room which is 22 ft. 6 ins. long and 15 ft. broad?

33. What would be the cost of varnishing a border 2 ft. wide all round th floor of a room which is 15 ft. long and 12 ft. broad at $1\frac{1}{4}$d. pe sq. ft.?

Find in sq. feet and sq. inches the area to be papered on the walls of room when :—

34. Length = 23 ft., breadth = 21 ft., height = 11 ft., and there is a doo 8 ft. by 3 ft. and a window 6 ft. by 4 ft.

35. Length = 22 ft. 6 ins., breadth = 20 ft. 6 ins., height = 10 ft. 6 ins., an there are 2 windows, each 6 ft. 6 ins. by 4 ft., and a door 7 ft. 6 ins by 3 ft. 6 ins.

36. Length = 15 ft., breadth = 14 ft., height = 9 ft. 4 ins., and there are door and window measuring 7 ft. by 3 ft. and 6 ft. by 4 ft. 4 ins respectively.

37. What length of wall-paper 2 ft. 3 ins. broad will be required for th walls of a room 21 ft. long, 18 ft. 6 ins. broad, and 9 ft. high Express the answer in yards and feet.

38. What is the cost of papering the walls of a room 15 ft. 6 ins. long 12 ft. 6 ins. broad, and 8 ft. high, with paper 2 ft. broad at $1\frac{1}{2}$d. pe foot?

39. Find the number of square feet to be lined with metal inside a cistern whose dimensions are 5 ft. long, 4 ft. 9 ins. broad, and 4 ft. 6 ins high.

40. If a cistern is to be lined with lead at $4\frac{1}{2}$d. per square foot, what wi be the cost, the length being 5 ft. 3 ins., breadth 5 ft., and heigh 4 ft. 9 ins.?

41. Find the cost of covering the sides and bottom of a box with leather a $1\frac{1}{4}$d. per square foot, when the box is 4 ft. 3 ins. long, 3 ft. broad and 3 ft. 3 ins. high. Answer to the nearest penny.

42. How many rectangular tiles $4\frac{1}{2}$ ins. by 6 ins. will be required to cove a floor 18 ft. long by 13 ft. 6 ins. broad? And what will be th cost at 3s. 4d. per dozen tiles?

3. A rectangular tank is 10 ft. 6 ins. long and 7 ft. 3 ins. wide, and contains 400 gallons of water. Find the depth of the water, having given that a gallon contains $277\frac{1}{4}$ cubic inches. Answer in inches.
4. What would be the cost of the material of a closed cubical box, 3 inches in thickness on all sides, whose interior is also cubical and contains 27 cubic feet, reckoned at 2s. 6d. per cubic foot of material?
5. What is the real area, in acres, of a field represented on a map drawn to a scale of $\frac{1}{1000}$ by a rectangle 5 inches long and $4\frac{1}{2}$ inches broad?
6. If, on a map drawn to a scale of $\frac{1}{1500}$, a rectangular field is represented by a rectangle $4\frac{1}{2}$ inches by $3\frac{2}{3}$ inches, what is the real area of the field in acres?
7. If a plot of land is represented on a map drawn to a scale of $\frac{1}{12000}$ by a square whose side is 3 inches, what is the area of the field in acres?
8. Find the height of a room 15 ft. long and 12 ft. broad, the walls of which, when covered with paper 2 ft. 3 inches wide at 3d. per yard, cost 18s.
9. The cost of papering the walls of a room 16 ft. 6 ins. long and 10 feet broad with paper 2 ft. wide, which is sold at 2s. 6d. per piece of 12 yards, is 17s. 6d.; what is the height of the room to the *nearest inch*?
10. If it cost 22s. to paper the walls of a room which is 20 ft. 3 ins. long and 15 ft. broad with paper 2 ft. 8 in. broad, the paper costing $2\frac{1}{2}$d. per yard, what is the height of the room to the *nearest inch*?
11. A box which measures 5·5 m. by 4·4 m. by 4·5 m. is coated all over with paint to a depth of 1 mm. What is the volume of paint used in cu. centimetres?
12. A box which is 7 m. 5 dm. long, 4 m. 5 cm. broad, and 5 m. high is covered all over with paint to a depth of ·0005 m. How many cu. centimetres of paint are used?
13. Given that the length of paper required to cover the walls of a room 85 dm. long, 65 dm. broad, and 4·4 m. high, is 240 m., what is the breadth of the paper in decimetres?
14. If the length of paper required to cover the walls of a room is 350 m. 5 dm., and the breadth of the strips is 5 dm. 4 cm., what is the area of the walls in sq. metres?

55. Given that the area of the four walls of a room is 138 sq. m. 35 sq. dm 25 sq. cm., and that the breadth of the strips of paper is 5·5 dm. what is the length of the paper required in metres?

56. The volume of a rectangular box is 15 cu. m., and the area of the bas is 6 sq. m. 25 sq. dm.; what is the height in decimetres?

57. Find the number of blocks of wood required to pave the floor of a hal 163·2 sq. metres in area, if each block is 3·2 dm. broad and 5·1 dm long.

58. How many blocks of paving-stone, each 1·3 metre long and 7·5 dm broad, will be required to pave a path whose area is 1950 sq. m.?

59. If the external dimensions of a closed wooden box are 8, 7, and decimetres respectively, how many sq. metres of wood, 5 millimetre thick, will be required for its construction?

60. Find the number of square decimetres of wood, 5 millimetres thick required for the construction of a lidless box which internally i 5 dm. long, 4 dm. broad, and 2 dm. deep.

Square Root—*continued*

If the square root of a number cannot be found by simple factors, the the Algebraic Method, depending on the formula $(a+b)^2 = a^2 + b(2a+b$ should be used.

Ex. 1. *Find the square root of* 4439449.

```
    4,43,94,49(2107      Answer.
    4
  ------------
41) 43
      41
  ----------
4207) 29449
      29449
```

[*Rough Check*: The number is approximately 4,000,000, and the squar root should be 2000 (approx.).]

Ex. 2. *Find the square root of* 8·678916.

```
8·67,89,16(2·946.   Answer.
4
49)467
 441
584) 2689
 2336
5886) 35316
 35316
```

***Rule.*—If n figures of a square root of a surd have been obtained, then $n-2$ more figures may be found by dividing the last remainder by the last divisor.**

Ex. 3. *Find the square root of* 8 *to* 5 *places of decimals.*

```
8(2·82842.   Answer.
4
48)400
 384
562) 1600
 1124
5648) 47600
 45184
  24160 . . . Here the remainder is 2416, and
  22592       the first four figures of the
   15680      square root having been ob-
   11296      tained, two more can be found
              by dividing 24160 by 5648.
```

Ex. 4. $\sqrt{65\frac{64}{81}} = \sqrt{\frac{5329}{81}} = \frac{73}{9} = 8\frac{1}{9}$.

Ex. 5. $\frac{1}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5}$; we must thus find the value of $\sqrt{5}$ and divide by 5.

Ex. 6. $\frac{3}{\sqrt{5}-\sqrt{2}} = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2} = \sqrt{5}+\sqrt{2}$;

the roots of 5 and 2 are then obtained separately and added.

Ex. 7. *Find the square root of* $54\frac{5}{11}$ *to 2 decimal places.*

$54\frac{5}{11} = 54{\cdot}4545 \ldots$ . . . (7·37 . . . . Answer.

$$\begin{array}{r} 49 \\ \hline 143)\ 545 \\ \quad 429 \\ \hline 1467)11645 \\ \quad 10269 \\ \hline \quad 1376 \text{ etc.} \end{array}$$

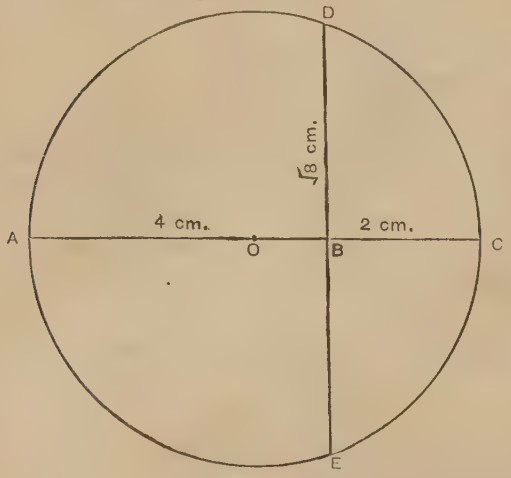
Ex. 8. *Given that the area of a rectangle is* 1183·36 *sq. metres and that the length is* 4 *times the breadth, find the perimeter.*

Let l metres be the breadth
$4l$,, ,, length.
Then $4l^2 = 1183{\cdot}36$
$l^2 = 295{\cdot}84$
$l = 17{\cdot}2$
$\therefore$ perimeter $= 2(l + 4l) = 10l = 172$ metres.

$$\begin{array}{r} 2,95{\cdot}84(17{\cdot}2 \\ 1 \\ \hline 27)195 \\ \quad 189 \\ \hline 342)\ 684 \\ \quad 684 \end{array}$$

Square roots may also be found geometrically :—

Ex. 9. *Find the square root of* 8.



(i.)
Measure $AB = 4$ cm.
and $BC = 2$ cm.

Bisect AC at O and describe a circle with centre O and radius OA.

From B draw the chord DBE perp. to AC.

Then
$DB.BE = AB.BC$
or $DB^2 = 4 \times 2 = 8$.
$\therefore DB = \sqrt{8}$.

By measurement, $DB = 2{\cdot}83$.

(ii.)

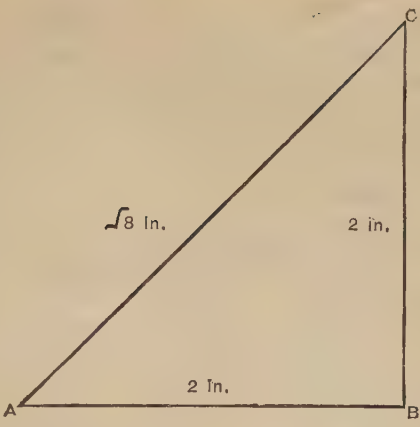
Measure $AB = 2$ inches.

Erect BC perp. to AB and equal to 2 inches.

Join AC.

Then $AC^2 = AB^2 + BC^2$
$= 2^2 + 2^2$
$= 8.$

$\therefore AC = \sqrt{8}.$



Exercise LVI.

Find the square roots of :—

1. 16129.
2. 137641.
3. 273529.
4. 185761.
5. 108241.
6. 124609.
7. 146689.
8. 208849.
9. 1423249.
10. 22581504.
11. 28·8369.
12. 5·774409.
13. 50·2681.
14. 3831·61.
15. 142·3249.
16. 2662·56.
17. 103·2256.
18. 342·25.
19. 5490·81.
20. 1·522756.
21. 2777·29.
22. 1231·3081.
23. ·0001841449.
24. ·01447209.
25. 1095·61.
26. ·000016056049.
27. ·05368489.
28. 3199·0336.
29. ·047961.
30. 10795·21.
31. 39·175081.
32. ·0576576144.
33. 2·5 to 3 places of decimals.
34. 135·246 to 3 places „
35. ·1079521 to 4 places „
36. 235·6 to 3 places „
37. ·001111111 to 5 places „
38. 32·14 to 4 places „
39. 3·1415926535 to 7 places of decimals.
40. ·00312481.
41. 3·9726523 to 5 places „
42. ·39726523 „ „
43. ·039726523 „ „
44. $11\frac{37}{45}$.
45. $9\frac{67}{121}$.
46. $4\frac{285}{529}$.

47. $27\frac{54}{361}$.
48. $9\frac{352}{841}$.
49. $2336\frac{1}{5}$.
50. $153\frac{7}{9}$ to 4 places of decimals.
51. $3083\frac{214}{225}$.
52. $1\frac{2}{3}$ to 4 places of decimals.
53. $76\frac{14}{17}$ to 5 „ „
54. $\frac{151329}{182329}$.
55. $34967\frac{2}{11}$ to 4 places „
56. $\frac{11}{5}$ to 7 places „

Calculate :—

57. $\frac{1}{\sqrt{8}}$ to 5 places of decimals.
58. $\frac{1}{\sqrt{7}}$ to 6 „ „
59. $\frac{1}{\sqrt{4}}$ to 4 „ „
60. $\frac{1}{\sqrt{10}}$ to 5 „ „
61. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ to 4 places of decimals
62. $\frac{\sqrt{7}}{\sqrt{7}-\sqrt{3}}$ to 4 „ „
63. $\frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}-\sqrt{7}}$ to 4 „ „
64. $\frac{3+\sqrt{5}}{3-\sqrt{5}}$ to 5 „ „

65. If the area of a square is 172225 square metres, what is the length, i metres, of one of the sides?
66. Given that a square field contains 18 acs. 2 rds. 460 sq. yds., what i the length of one of the sides expressed in yards?
67. If a square field measures 37 acs. 1 rd. 335 sq. yds., how many yard will one of the sides contain?
68. Find the number of yards in the side of a square field which measure 9 acs. 2 rds. 245 sq. yds.
69. A rectangular field covers 2 acs. 1136 sq. yds., and the length is fou times the breadth. Find the breadth in yards.
70. What length of fencing (in metres) will be required to enclose a squar field whose area is 1806·25 square metres?
71. Find the perimeter of a square patch of ground whose area is 930·2 square metres. Express the answer in hectometres.
72. If the length of a field is three times the breadth, and the area 7410·27 square metres, find the length in metres.
73. Find the length of the diagonal of a square, each side of which is 30 metres. Give the answer in metres to one decimal place.
74. Find the length of the diagonal of a square, each side of which is 32 metres. Answer in metres to 1 place of decimals.

Find geometrically to 2 decimal places :—

75. $\sqrt{7}$.
76. $\sqrt{10}$.
77. $\sqrt{13}$.
78. $\sqrt{15}$.
79. $\sqrt{8}$.
80. $\sqrt{11}$.
81. $\sqrt{14}$.
82. $\sqrt{17}$.
83. $\sqrt{19}$.

Unitary Method—*continued*

Ex. 1. *If* 3 *men and* 4 *boys do a piece of work in* 15 *days, how long will it take* 10 *men and* 15 *boys, supposing that* 1 *man does as much work as* 2 *boys?*

3 men + 4 boys = 10 boys
10 men + 15 boys = 35 boys
10 boys do the work in 15 days
$\therefore$ 35 ,, ,, ,, $\frac{15 \times 10}{35}$ days.
$\therefore$ Time $= \frac{15 \times 10}{35} = \frac{30}{7} = 4\frac{2}{7}$ days.

Ex. 2. *If* 6 *men and* 12 *boys do a piece of work in* 10 *days, which* 21 *men and* 37 *boys could do in* 3 *days, how long would it take* 10 *men and* 15 *boys?*

6 men + 12 boys do work in 10 days.
$\therefore$ 60 men + 120 boys ,, ,, ,, 1 day.
21 men + 37 boys ,, ,, ,, 3 days.
$\therefore$ 63 men + 111 boys ,, ,, ,, 1 day.
Thus 60 men + 120 boys = 63 men + 111 boys.
$\therefore$ 3 men = 9 boys.
1 man = 3 boys.
$\therefore$ 6 men + 12 boys (from first statement) = 30 boys
and 10 men + 15 boys (,, last ,,) = 45 boys.
30 boys do work in 10 days.
$\therefore$ 45 ,, ,, ,, $\frac{10 \times 30}{45}$ days.
$\therefore$ Time $= \frac{20}{3} = 6\frac{2}{3}$ days.

Ex. 3. *If* 10 *men mow* 12 *acres in* 3 *days, working* 9 *hours a day, how many hours a day must* 15 *men work to mow* 20 *acres in* 4 *days?*

10 men mow 12 acres in 3 days, working 9 hours a day
15 ,, 20 ,, 4 ,, ? ,,

$$\text{No. of hours a day} = \frac{\overset{3}{\cancel{9}} \times \overset{5}{\cancel{10}} \times \overset{5}{\cancel{20}} \times 3}{\underset{3}{\cancel{15}} \times \underset{1}{\cancel{12}} \times \underset{1}{\cancel{4}}} = 7\tfrac{1}{2}.$$

[If 15 men are engaged on the same field for the same number of days, *they will not have to work so many hours a day* as 10 men; therefore we *multiply* by the *smaller* number 10 and divide by 15.

If a certain number of men have to mow 20 acres instead of 12 acres in the same number of days, *they must work a greater number of hours a day*; therefore we *multiply* by the *greater* number 20 and divide by 12.

If the men may mow the field in 4 days instead of 3 days, *they need not work so many hours a day*; and therefore we *multiply* by the *smaller* number 3 and divide by 4.]

Exercise LVII.

1. If 5 men *or* 7 women can do a piece of work in 74 days, in what time will 7 men *and* 5 women do the same piece of work?
2. If a garrison of 37 men have enough food to last them 18 days, and are then reinforced by 8 men, how long will the food last?
3. Given that 22·4 litres of ammonia gas at normal temperature and pressure weigh 17 grams, express in grams, correct to two places of decimals, the weight of 55 litres at same temperature and pressure.
4. If 22·4 litres of marsh gas at normal temperature and pressure weigh 16 grams, find in grams, correct to two places of decimals, the weight of 72 litres at the same temperature and pressure.
5. If it cost as much to feed 3 men as to feed 4 boys, and for 3 boys the cost is found to be 19s. $2\frac{1}{4}$d. per week, how much per week will it be for 51 men?

6. A man reaches a town in 18 days by walking 9 hours a day; how many days would he have taken had he walked 12 hours a day?

7. If 1 lb. of gold be coined into £46, 14s. 6d., what is the weight of a sovereign?

8. If 8 men *or* 12 boys can build a wall in 7 days, how long will it take 8 men *and* 9 boys?

9. If 14 men and 40 boys can do a piece of work in 100 days which 18 men and 80 boys could do in 60 days, how long will it take 216 men and 585 boys?

10. If 7 men and 12 women do as much work in 80 days as 13 men and 18 women do in 48 days, how long will it take 51 men and 36 women to do the same piece of work?

11. If 25 ponies eat a certain amount of hay in 4 days, in how many days will 55 ponies eat the same quantity?

12. If a person after paying a tax of 10d. in the £ has £728, 6s. 8d. remaining, what had he at first?

13. If 63 guns of equal weight together weigh 201 tons 4 cwts. 2 lbs. 3 ozs., what is the weight of 1 gun in ounces?

14. If 25 lambs consume a certain number of turnips in 51 days, in how many days will 42 lambs consume the same number of turnips?

15. If 4 grains are equal to ·2592 grams, how many grams will there be in 9 grains?

16. Given that 5 kilograms = 11·023 pounds, how many pounds will there be in 18 kilograms?

17. 5 tons are equivalent to 5083·25 kilograms; how many kilograms are equivalent to 59 tons?

18. It is known that 4 cubic feet of water at 4° C. weigh 249·6 pounds, what will be the weight in pounds of 21 cu. feet at the same temperature?

19. Given that 3 pounds of water at 4° C. occupy ·048 cu. feet, what will be the volume of 29 pounds of water at the same temperature?

20. If 22·4 litres of oxygen at the normal temperature and pressure weigh 32 grams, what will be the weight of 70 litres of oxygen at the same temperature and pressure?

21. Calculate the income tax on £722 at 1s. 2d. in the £.

22. If the railway fare for 245 miles is £1, 10s. 7½d., what will be the fare for 627 miles?

23. How much must be given for 13 articles at the rate of £3, 16s. 6d. for 3 articles?

24. If a sovereign weighs 5 dwts. 8 grs., what is the value of 106 lbs. 9 ozs. 16 dwts. of the same metal?

25. If 3 men do a piece of work in 12 days, working 7 hours a day, how long will it take 7 men working 9 hours a day?

26. A trench 87 yards long can be dug by 18 men in 7 days; how many men would it require to dig a trench 145 yards long in 10 days?

27. If the carriage for 2 cwts. 1 qr. 8 lbs. for 100 miles is 4s. 4d., what weight could be carried 200 miles for £1, 11s. 8d.?

28. If 10 people can mow a field 2 furlongs long and 200 yards wide in 6 hours, in how many hours will 12 people mow a similar field 420 yards long and 212 yards wide?

29. If 15 people can reap a field of 750 sq. poles in 7 hours, in how many hours will 39 people reap a field of 520 sq. poles?

30. If by working 6 hours a day 5 men can do a piece of work in 14 days, how many hours a day must 18 men work in order to accomplish the same piece of work in 3⅓ days?

31. If 225 lbs. can be carried 130 miles for 5s. 5d., what should be the charge for carrying 5 cwts. 10 lbs. a distance of 150 miles?

32. If 33 men dig 121 yards of a trench in 21 days, in how many days will 52 men dig 200 yards?

33. If the fares for 12 people for 150 miles come to £8, 15s., how many miles could 21 people ride for £11?

34. The rent of a field of 6 acs. 1 rd. 20 poles for 2½ years is £22; for how many years will the rent of 950 poles come to £15?

35. If 3 horses can plough a field of 10 acres in 2 days, in how many days will 5 horses plough 12 acres?

. If 12 animals are fed for 5 days for £3, 4s., what will be the cost of feeding 14 animals for 30 days?

. The rent of 9 acres 10 sq. poles being £40, 4s. for 6 months, for how many sq. poles will £50, 17s. be the rent for 15 months?

. If 7 oxen eat an acre of grass in 6 days, how long will it take 17 oxen to eat 34 acres?

. If a beam 16 feet long, $2\frac{1}{2}$ feet broad, and 8 inches thick weigh 1280 pounds, what must be the length in feet of a beam of the same material, whose breadth is $3\frac{1}{4}$ feet, thickness $6\frac{1}{2}$ inches, and weight 2028 pounds?

. The moon describes, relatively to the earth, a circle of 240,000 miles radius in $27\frac{1}{3}$ days; find to the *nearest minute* the time she takes to travel over a space equal to her own diameter (2160 miles), the circumference of a circle being $3\frac{1}{7}$ times its diameter.

. A contractor undertakes to do a piece of work in 38 days. By employing 60 men on it, he does $\frac{2}{3}$ of it in 22 days; how many additional men must he employ to finish it in time?

. How many days of 12 hours each will 14 men require to perform a piece of work which would employ 70 men for 30 days of 8 hours each, supposing that they all work at the same rate?

Contracted Work

[*This section may be deferred if it is thought advisable.*]

By saying that ·1492 is *correct* to 4 decimal places, we mean that it may anything less than ·14925, but not less than ·14915.

Thus the *limits of error* are ± ·00005.

In writing down a decimal correct to a certain number of places, if the t figure after the required number of decimal places is 5, 6, 7, 8, or 9, we **l 1 to the last figure required;**

If the first figure after the required number of decimal places is 1, 2, 3, 4, we omit it.

Thus, ·15957 *correct to* 4 *decimal places* is ·1596.

·20352 „ „ „ ·2035.

Addition and Subtraction

Ex. 1. *Add together* 5·927856, 6·007259, 8·0307277 *and give the answe correct to* 4 *places of decimals.*

5·9278	56
6·0072	59
8·0307	27
19·9658	42

∴ Sum = 19·9658.

[Retain to the right of the decimal point 2 columns more than the numbe of figures required in the answer.]

Ex. 2. *Find the difference between* 10·528741 *and* 12·0072994 *correct t* 4 *decimal places.*

12·0072	99
10·5287	41
1·4785	58

∴ Diff. = 1·4786.

Multiplication

Ex. 3. *Multiply* 5·23978586 *by* 17·32 *correct to* 4 *places of decimals.*
[*Rough Check*: $5 \times 17 = 85$.]

52·397859
1·732
52·397859
36·678501
1·571936
·104796
90·7531 Answer.

[i. Alter the two numbers so that there is only *one figure to the left* the decimal point in the *multiplier*.

ii. In order to obtain the product correct to 4 places, we retain (4 + 2 figures to the right of the decimal point in the multiplicand and each partia product (adding ciphers if necessary), the last figure of the multiplican being corrected if necessary.

iii. To obtain each partial product after the first, one figure may b struck off the multiplicand, retaining it mentally, to see if there is anythin to carry ; any figure which is carried, should be *corrected*.

iv. In performing the final addition, the last two columns are adde merely in order that the last figure written down in the product may b *corrected*.]

Ex. 4. *Find correct to 3 decimal places the product of* ·5403257, 832·5 *and* ·0724.

Product = ·5403257 × 8·325 × 7·24.

[*Rough Check*: ·5 × 8 × 7 = 28.]

·5403257	
8·325	
4·3226056	
·1620977	
·0108065	
·0027016	
4·498321	
7·24	
31·48747	
·89964	
·17993	
32·567	Answer.

Division

i. Work on the assumption that *one place more than asked for is to be found.*

ii. As in ordinary division, arrange so that there is *one figure to the left of the decimal point in the divisor.*

iii. *Retain as many figures* to the right of the decimal point in the *dividend* (correcting the last figure) as are required to the right of the decimal point in the *quotient*, adding ciphers if necessary.

iv. *Divide in the ordinary way until* there are no more figures to bring down from the dividend, then the right-hand figure of the divisor is struck off (retaining it mentally, however, to see if there is any number to carry), and so on for the succeeding steps.

v. A *figure carried* from a digit which has been struck out in the divisor should be *corrected.*

Ex. 5. *Divide* 5623 *by* 1547 *correct to* 4 *places of decimals.*

[*Rough Check*: Quotient = 6000 ÷ 2000 (approx.) = 3.]

```
                3·63478
              ----------
1·5471)5·62300
       4·641
       ------
        9820
        9282
        ----
         5380
         4641
         ----
          739 ..... At this step, there being no
          619       more figures to bring down, the
          ---       7 of the divisor is struck out;
          120       the 4 of the divisor struck out
          108       at the next step, and so on.
          ---
           12
           12
           --
```

∴ Quotient = 3·6348 (correct to 4 places).

Ex. 6. *Divide* 1·378967 *by* 238·9605 *correct to* 4 *significant figures.*

[*Rough Check*: Quotient = 1 ÷ 200 = ·01 ÷ 2 = ·005.]

```
                ·0057707
              ------------
2·389605)·0137897
         119480
         ------
          18417
          16727
          -----
           1690
           1672
           ----
             18
             16
             --
```

∴ Quotient = ·005771 (correct to 4 significant figures).

[Dividing ·01 by 2, we see that the quotient starts with 2 ciphers, and thus we shall require 6 figures in the answer, and must work on the assumption that 7 are needed.

It will be noticed that the final figures 0 and 5 of the divisor are not required for the first partial product, except to see what number to carry.]

Ex. 7. *Find the value of* $\dfrac{5{\cdot}82347 \times 9{\cdot}23643}{4{\cdot}12935}$ correct to 3 places.

$$\text{Exp.} = \frac{5{\cdot}82347 \times 9{\cdot}23643}{4{\cdot}12935}.$$

[*Rough Check*: $\text{Exp.} = \dfrac{6 \times 9}{4} = 13$ (approx.).]

[Since the final answer is to be correct to 3 places, the numerator must be obtained correct to 4 places, and thus 6 columns to the right of the decimal point retained in the multiplication.]

$$
\begin{array}{r}
5{\cdot}823470 \\
9{\cdot}23643 \\
\hline
52{\cdot}411230 \\
1{\cdot}164694 \\
{\cdot}174704 \\
{\cdot}034940 \\
{\cdot}002329 \\
{\cdot}000175 \\
\hline
4{\cdot}12935)53{\cdot}7881 \quad (13{\cdot}0258 \\
41\ 2935 \\
\hline
12\ 4946 \\
12\ 3881 \\
\hline
1065 \\
828 \\
\hline
239 \\
206 \\
\hline
33 \\
33
\end{array}
$$

$\therefore$ Quotient = 13·026 (correct to 3 places).

Exercise LVIII.

Find the value of :—

1. $3{\cdot}24678 + 5{\cdot}20274 + 8{\cdot}23456$ correct to 3 places.
2. $51{\cdot}072345 + 4{\cdot}3015789 + 10{\cdot}007254$ „ „
3. $32{\cdot}035732 + 81{\cdot}1230721 + 3{\cdot}002415$ „ 4 places.
4. $15{\cdot}42\dot{1} + 17{\cdot}02\dot{1}4 + 13{\cdot}00\dot{2} + 2{\cdot}51$ „ „
5. $32{\cdot}01\dot{3} + 30{\cdot}02\dot{7} + 40{\cdot}85\dot{6}$ „ „
6. $717{\cdot}301 + 50{\cdot}209 + 7{\cdot}53\dot{2} + 4{\cdot}5$ „ „

7. $512{\cdot}32472 - 39{\cdot}31215$ correct to 3 places.
8. $42{\cdot}0795 \ldots 30{\cdot}003423$ „ „
9. $771{\cdot}0023491 - 7{\cdot}32415$ „ „
10. $742{\cdot}32\dot{5} - 700{\cdot}5\dot{1}$ „ „
11. $882{\cdot}7\dot{4}\dot{7} - 32{\cdot}5\dot{2}\dot{1}$ „ „
12. $427{\cdot}82 - 52{\cdot}\dot{3}$ „ „
13. $2145{\cdot}\dot{8}9\dot{7} - 51{\cdot}2$ „ „
14. $\frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \ldots$ so that the error may be less than ·00001.
15. $\frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5} \ldots$ correct to 4 places.

Multiply :—

16. 1·528796 by 3, 15 and 18 correct to 3 places.
17. 11·30279 by 7·432 „ „
18. 75·08456 by 10·21924 „ „
19. 55·72051 by 8·20307 „ „
20. 52·43789 by 7·20051 „ „
21. 52·427956 by 3·0715 „ 4 places
22. 83·324578 by ·01234 „ „
23. 55·9276543 by 10·321 „ „
24. 501·245976 by 3·079 „ „

Find the value of :—

25. $23{\cdot}72561 \times 3{\cdot}425 \times 7{\cdot}201$ correct to 3 places.
26. $85{\cdot}07314 \times 5{\cdot}035 \times {\cdot}342$ „ „
27. $52{\cdot}03071 \times {\cdot}0231 \times 3{\cdot}027$ „ „

Divide :—

28. 57·302147 by 3·2, 5·279 and 4·3105 correct to 3 places.
29. 52·40075 by 7·241, 5·9072, and 6·30455 „ „
30. 525·70234 by 5·72341 correct to 4 places.
31. 5·29705 by ·02391 „ „
32. 62·002957 by 5·20754 „ „
33. 182·9025 by 21·0705 „ „
34. 92·000567 by ·07182 „ „
35. 50·709801 by 5·307245 „ 5 places.

Find the value of :—

36. $\dfrac{32{\cdot}47851 \times 5{\cdot}70251}{5{\cdot}42031}$ correct to 3 places

37. $\dfrac{53{\cdot}05045 \times 3{\cdot}5204}{{\cdot}30455}$ " "

38. $\dfrac{500{\cdot}3425 \times {\cdot}0324}{25{\cdot}424}$ " "

39. Calculate, correct to 5 decimal places,

$$\frac{14{\cdot}52}{83{\cdot}1} + \frac{1{\cdot}575}{13{\cdot}85}.$$

40. Express $\dfrac{1}{3{\cdot}1416}$ as a decimal fraction true to the nearest thousandth. Use your result to find to the nearest inch the diameter of a circle which has a circumference of 87·97 inches. (The circumference of a circle is 3·1416 times the diameter.)

41. Knowing the number of pounds in a cubic inch of a substance, you can find the number of kilograms in a cubic centimetre by multiplying by $0{\cdot}4536 \times (2{\cdot}54)^{-3}$. Express this multiplier as a decimal correct to 3 places. If steel weigh 488 lbs. per cubic foot, how many kilograms per cubic centimetre does it weigh?

42. A kilogram is the weight of a cubic decimetre of water, and a decimetre is 3·937 inches. It being given that a cubic foot of water weighs a thousand ounces, express the kilogram, in pounds avoirdupois, correctly to two places of decimals.

43. An iron bar is 117 centimetres long, and its cross section is a square of which the side measures 9 millimetres. Find its weight to the nearest gram, supposing the iron to weigh 7·6 grams per cubic centimetre.

44. The French metre is 3·281 English feet, the Rhenish foot is 12·356 English inches. Express 4·567 metres in terms of Rhenish feet correct to 3 places of decimals.

45. Express $\sqrt{\dfrac{0{\cdot}00456 \times 0{\cdot}987}{6{\cdot}54}}$ as a decimal correct to three significant figures.

46. Express $\sqrt{\frac{0{\cdot}0132 \times 0{\cdot}543}{7{\cdot}65 \times 0{\cdot}0301}}$ as a decimal correct to three significant figures.

47. Assuming that the earth's polar circumference is 40,000,000 metres, and that a metre is 39·3708 inches, calculate the earth's polar circumference in miles, correct to the nearest unit.

48. If a sovereign is worth 25f. 25c., and a metre equals 39·3708 inches, find to the nearest centime the value in francs and centimes of a metre of cloth which is worth 5s. per yard.

Examples on Mensuration of Circle and Cylinder

Exercise LIX.

[*These examples may be deferred.*]

Circumference of $\odot = 2\pi r$; Area of circle $= \pi r^2$;

Area of curved surface of cylinder $= 2\pi rh$; Volume of cylinder $= \pi r^2 h$

Assume $\pi = 3\frac{1}{7}$.

1. Find the circumferences of circles with radii 5 inches, 7 cm., 7·2 cm. (correct to nearest mm.) respectively.
2. Find the areas of circles with radii 8 feet, 12 metres (correct to nearest sq. dm.), 5·6 cm. respectively.
3. Find the area of the curved surface of a cylinder when
   (i) $r = 5$ cm., $h = 10{\cdot}3$ cm. (correct to nearest sq. mm.).
   (ii) $r = 4{\cdot}2$ m., $h = 41{\cdot}6$ m.
   (iii) $r = 1{\cdot}2$ m., $h = 24{\cdot}2$ dm. (correct to nearest sq. dm.).
4. Find the volume of a cylinder when
   (i) $r = 4{\cdot}5$ cm., $h = 10{\cdot}2$ cm. (correct to nearest cu. mm.).
   (ii) $r = 21{\cdot}3$ m., $h = 100{\cdot}4$ dm. (,, ,, cu. dm.).
   (iii) $r = 1{\cdot}42$ cm., $h = 51{\cdot}2$ mm. (,, ,, cu. mm.).
5. Find (approximately) the radii of circles the circumferences of which are (i) 200 metres, (ii) 500 feet, (iii) 2·46 yards.
6. Find to the nearest ounce the weight of a cylindrical bar of metal 5 feet long and 1 inch in diameter, if 1 cu. foot of the same metal weighs 7900 ounces. ($\pi = 3{\cdot}142$.)

7. If the 28 in. wheels of a bicycle made 4000 revolutions each, how far did the machine travel? (Answer to the nearest yard.)
8. Find the area (correct to the nearest sq. foot) of a path 5 feet wide surrounding a circular lake of diameter 316 feet.
9. Find the cost (to the nearest shilling) of paving a circular courtyard at 6s. 6d. per sq. yard, if the diameter is 55 feet.
10. What is the cost of fencing a circular field of diameter 215 metres at 3f. 50c. per 10 metres?
11. Find the weight of a cylindrical stick of sulphur of length 3·5 dms. and diameter 2·5 cms., if 1 cu. cm. weigh 2 grams.
12. Find the weight of water (in ounces) in a cylindrical vessel 2 feet deep and 18 inches in diameter, if 1 cu. foot of water weighs 1000 ounces.
13. Find the area of a circular ring whose inner and outer radii are 7 and 13 feet respectively. (Answer in sq. feet.)
14. How many cu. feet of metal are there in a hollow cylindrical pipe 30 feet long, if the external diameter is 4 feet and the thickness of the metal 1 inch?
    Find in sq. feet the area of the inside of the pipe.
15. Find the total area of the surface of a cylinder of radius 5 inches and height 2 feet. (Answer in sq. inches.)
16. If the driving wheel of a locomotive is 7 feet in diameter and it makes 145 revolutions per minute, what is the rate of the train in miles per hour?
17. A cylindrical tank contains 5700 gallons. If the diameter of the base is 6 ft. 3 in., what is the depth? 1 cu. ft. $= 6\frac{1}{4}$ gallons. (Answer to the nearest inch.)
18. Find, to the nearest kilogram, the weight of metal in a hollow iron pipe 25 metres long, if the external diameter is 9·4 dm. and the thickness of the metal 2·5 cms. 1 cu. cm. of metal weighs 7·7 gms.
19. A subterranean railway station is cylindrical; it is 35 feet in diameter and 60 yards long. Find the weight of earth excavated in making it and the weight of the iron tube required to case it. Assume that 1 cu. yd. of soil weighs 1 ton, that the iron casing is 1 inch thick, and that 1 cu. foot of iron weighs 450 pounds. (Answer to the nearest ton.)

Ratio

Ex. Find in its simplest form the ratio of 12 cwts. 2 qrs. to 1 ton 10 cwts.

$$\text{Ratio} = \frac{12 \text{ cwts. } 2 \text{ qrs.}}{1 \text{ ton } 10 \text{ cwts.}} = \frac{50 \text{ qrs.}}{120 \text{ qrs.}} = \frac{5}{12}.$$

Exercise LX.

Express in their simplest form the ratio of :—

1. 150 to 270.
2. 60 to 84.
3. $1\frac{2}{3}$ to $5\frac{2}{3}$.
4. $5\frac{3}{7}$ to $2\frac{1}{14}$.
5. ·33 to ·66.
6. ·25 to 6·25.
7. 19·85 m. to 277·9 dm.
8. 3773 mm. to 4·116 m.
9. 19·75 Dm. to 4·345 Hm.
10. 6s. 5d. to 8s. 3d.
11. £2, 3s. 4d. to £2, 14s. 2d.
12. £17, 10s. 0d. to £18, 19s. 2d.
13. 4 lbs. 9 ozs. 15 dwts. to 6 lbs. 8 ozs. 17 dwts.
14. 1 ton 9 cwts. 1 qr. 24 lbs. to 1 ton 12 cwts. 16 lbs.
15. Find x, if $5 : x = 7 : 8$.
16. Find y, if $y : 7 = 8 : 9$.
17. Find z, if £1, 10s. 0d. : £3 = 9 : z.
18. Find the ratio of the area of a circle of radius 5 cms. to one of 1·2 dms.
19. Find the ratio of the area of the curved surface to that of one of the circular ends of a cylinder of radius 7 inches and height 10 inches.
20. Find x, if $5 : 8 =$ £2, 15s. 0d. : x.

Proportional Parts and Partnership

Ex. 1. *Divide* £81 *among* 3 *people so that their shares may be to one another as* 4, 3, 2.

Since $4 + 3 + 2 = 9$,

∴ 1st person receives $\frac{4}{9}$ of £81 = £36.

2nd „ „ $\frac{3}{9}$ of £81 = £27.

3rd „ „ $\frac{2}{9}$ of £81 = £18.

Ex. 2. *Divide £56 among 3 people so that A may have twice as much as B, and B twice as much as C.*

A's share = B's share × 2 = C's share × 4.
B's share = C's share × 2.
∴ The shares have the same ratio as the numbers 4, 2, 1;
and since 4 + 2 + 1 = 7,
∴ A's share = $\frac{4}{7}$ of £56 = £32.
B's share = $\frac{2}{7}$ of £56 = £16.
C's share = $\frac{1}{7}$ of £56 = £8.

Ex. 3. *A puts £500 into a business for 3 years, and B £700 for 5 years; how should they divide the profits of £2500 at the end of the time?*

The shares are proportional to 500 × 3 and 700 × 5, *i.e.* to 3 and 7.
∴ A's share = $\frac{3}{10}$ of £2500 = £750.
B's share = $\frac{7}{10}$ of £2500 = £1750.

Exercise LXI.

1. Divide £1 [illegible] to 2 parts proportional to 3 and 7.
2. Divide £5[illegible] to 3 parts proportional to 20, 33, and 51.
3. Divide 732 into 3 parts proportional to 9, 28, and 24.
4. Divide £[illegible]7s. into an equal number of half-crowns and shillings.
5. The sum of £[illegible]s divided up amongst 3 people in the ratio of 3, 5, and 8; how much does each person get?
6. Divide £104 among 3 people so that the amount they get may be in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
7. If one person receives 2s. for every 5s. received by another person, how will a sum of £16, 2s. be divided between them?
8. A body of 2400 men is to be raised from 3 towns whose populations are 11955, 16737, and 28692 respectively; find the number which each town should provide.
9. Four houses have to pay for the pathway being paved; their frontages are 15 feet, 16 feet, 18 feet, and 20 feet respectively. If the total cost is £39, 19s. 3d., how much will each of them have to pay?

10. Gun-metal contains 9 parts of copper to every 1 part of tin. How many pounds of copper will there be in 12 cwts. 26 lbs. of gun-metal?

11. Britannia-metal contains tin, copper, and antimony in the proportion of 140, 3, and 9; how many pounds of each of these metals will there be in 14 cwts. 3 qrs. 20 lbs. of Britannia-metal?

12. Lead, antimony, and tin are present in type-metal in the ratio of 75, 20, 5; how many pounds of lead will there be in 6 cwts. 1 qr. 25 lbs. of the type-metal?

13. A certain sum of money is divided amongst three people in the proportion of 2, 3, and 5; the first person receives 7s. 7d. What is the total sum?

14. Divide £523, 10s. among 3 people, A, B, and C, so that A may get $\frac{1}{3}$ of what B receives, and C may receive as much as A and B together.

15. Two persons, A and B, enter into partnership with capitals of £500 and £700 respectively; their profits for the first year amount to £360. What portion should each of them receive?

16. Three people who own a business make a profit of £430, 10s.; how should this be divided, if they have put in £500, £600, and £1000 respectively?

17. Three people who invest £1000, £1200, and £1500 respectively, enter into partnership, and the profits at the end of the year are £661 16s. 9d.; what will be the share of each?

18. Divide £25, 7s. 6d. between 4 men, 3 women, and 2 boys, giving each woman a fourth less, and each boy a half less, than the share of each man.

19. Three graziers, A, B, C, agree to hold a pasture in common. A puts in 8 oxen for 10 months; B 7 oxen for 12 months; C 20 oxen for 4 months. The rent is £66 per annum. How much should each contribute?

20. Three people, A, B, C, enter into partnership. A supplies £1000 for 12 months, B £1500 for 6 months, and C £2000 for 3 months. The profits for 1 year come to £360; find the share of each.

1. Three people, A, B, and C, go into a business. A puts in £1500 at the beginning of the year; after 3 months B joins with £1800; after another 2 months C joins with £2000. The profits for the year are £253, 1s.; how should they be divided?

2. A, B, and C rent a pasturage for £12, 8s.; A puts in 15 cows for 2 months, B 20 for 3 months, and C 25 for 6 months. How much will each person have to pay?

3. A, B, and C rent a field. A puts in 12 cows for 3 months, B 15 cows for 6 months, and C 18 cows for 4 months. A's share is £12; what do B and C pay?

Percentages, Etc.

Ex. 1. *Find the value of* 12% *of* £17, 10s. 0d.

$$\text{Value} = £17\tfrac{1}{2} \times \frac{12}{100} = £\frac{\overset{7}{35} \times \overset{3}{12}}{\underset{1}{2} \times \underset{5}{100}} = £\frac{21}{10} = £2,\ 2\text{s. } 0\text{d.}$$

Ex. 2. *What per cent. of* 3 tons *is* 5 cwts.?

On 60 cwts. we reckon 5 cwts.
,, 100 ,, ,, ,, ?

$$\therefore \text{Rate } \% = \overset{5}{\cancel{5}} \times \frac{100}{\underset{12}{\cancel{60}}} = \frac{25}{3} = 8\tfrac{1}{3}.$$

Alternative Method: $60 \times \frac{\text{rate}}{100} = 5.$ $\therefore$ rate $= \frac{5 \times 100}{60} = 8\tfrac{1}{3}.$

Ex. 3. 7 *is* 30 *per cent. of what number?*

30 is reckoned on 100.
7 ,, ,, ,, ?

$$\therefore \text{Number} = 100 \times \frac{7}{30} = \frac{70}{3} = 23\tfrac{1}{3}.$$

Alternative Method: Number $\times \frac{30}{100} = 7.$ $\therefore$ Number $= \frac{7 \times 100}{30} = 23\tfrac{1}{3}.$

Ex. 4. *A broker purchases £12000 of goods; what will be his commission at $\frac{1}{8}\%$?*

$$\text{Commission} = \tfrac{1}{8}\% \text{ of } £12000.$$
$$= £12000 \times \frac{1}{800} = £15.$$

Ex. 5. *Find the value of 5·75% of £41, 13s. 8¼d. to the nearest penny.*

$$\text{Value} = £41{\cdot}684 \times \frac{5{\cdot}75}{100} = £{\cdot}41684 \times 5{\cdot}75$$
$$= £2{\cdot}397$$
$$= £2,\ 7\text{s. } 11\text{d. (correct to nearest penny)}$$

·41684
5·75
2·08420
·29179
·02084
2·397

Ex. 6. *From 1889 to 1902 the world's output (in metric tons) of iron ha increased from 26,000,000 to 42,669,000. Find the increase correct to 1 place of decimals.*

On 26,000,000 metric tons the increase is 16,669,000.
,, 100 ,, ,, ?
$$\therefore \text{Increase } \% = 16{,}669{,}000 \times \frac{100}{26{,}000{,}000} = \frac{166{\cdot}69}{2{\cdot}6}.$$
$$= 64{\cdot}1 \text{ (correct to 1 place).}$$

Ex. 7. *Find the error % in taking $1 + \frac{1}{100}$ as the value of $\frac{1}{1 - \frac{1}{100}}$.*

$$\text{True value} = \frac{1}{1 - \frac{1}{100}} = 1\tfrac{1}{99} = 1{\cdot}0101 \ldots\ldots$$
$$\text{Approximate value} = 1 + \frac{1}{100} = 1{\cdot}01.$$
$$\therefore \text{On the true value of } 1{\cdot}0101 \ldots\ldots \text{ the error is } {\cdot}000101 \ldots\ldots$$
$$,, \quad ,, \quad 100 \quad ,, \quad ?$$
$$\therefore \text{Error } \% = {\cdot}000101 \times \frac{100}{1{\cdot}0101 \ldots\ldots} = \frac{{\cdot}0101 \ldots\ldots}{1{\cdot}0101 \ldots\ldots}.$$
$$= {\cdot}01$$

Exercise LXII.

[*Examples* 1 *to* 5 *should be taken orally.*]

1. Give the following rates per cent. as fractions in their lowest terms :—

5, 10, 30, 25, 40, $2\frac{1}{2}$, $7\frac{1}{2}$, 2C, $33\frac{1}{3}$, $16\frac{2}{3}$, $12\frac{1}{2}$.

2. Express the following fractions as rates per cent. :—

$\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{5}$, $\frac{3}{4}$, $\frac{1}{40}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{6}$.

3. Find :—

(*a*) 10% of £240. (*β*) 5% of £360. (*γ*) $2\frac{1}{2}$% of £320.

(*a*) $33\frac{1}{3}$% of 300 ft. (*β*) $12\frac{1}{2}$% of 320 lbs. (*γ*) 25% of £48, 4s. 0d.

(*a*) 20% of 10·5 metres. (*β*) 30% of 17·2 grams. (*γ*) 40% of 1·2 litres.

4. Find the value of :—

10% of £45.

4% of £125.

3% of £55.

20% of 155.

$4\frac{1}{4}$% of £73, 15s., *correct to the nearest penny.*

$6\frac{1}{2}$% of £22, 15s. 9d., „ „

6·25% of £51, 13s. $9\frac{1}{2}$d., „ „

$3\frac{1}{2}$% of £72, 13s. 4d., „ „

3·75% of £82, 19s. 8d., „ „

4·375% of £32, 5s. $7\frac{3}{4}$d., „ „

5·75% of £32, 11s. $8\frac{3}{4}$d., „ „

8% of 4 cwts. 2 qrs. 21 lbs.

4% of 3 hectolitres 25 litres. Express answer in litres.

13% of 45 kilograms. Express answer in kilograms.

10. $2\frac{1}{2}$% of £50.

11. $7\frac{1}{2}$% of 55.

12. $2\frac{1}{2}$% of £18, 10s.

5. What percentage of :—

525 is 21 ?

686 is 49 ?

520 is $2\frac{3}{5}$?

£7, 10s. is 6s. ?

7 cwts. 16 lbs. is 1 qr. 12 lbs. ?

28. 18 yds. 2 ins. is 1 yd. 5 ins. ?

29. 22 yds. 8 ins. is 2 ft. 4 ins. ?

30. £2, 7s. 8d. is 1s. 1d. ?

31. £577, 10s. is £30, 16s. ?

If the recorded sunshine is 81 hours out of a possible 450, how much per cent. is this ?

33. What percentage of 3 ozs. 12 dwts. 16 grs. is 18 dwts. 4 grs. ?
34. If on 20 quarters there is a loss of 64 gallons, what is the loss pe cent. ?
35. What percentage of £1665, 9s. 4½d. is £33, 6s. 2¼d. ?
36. What percentage of an hour is 3 minutes ?
37. 18 is 33⅓ per cent. of what number ?
38. 6 is 6¼ per cent. of what number ?
39. 9 is 4 „ „
40. 3·6 is 5 „ „
41. 2 qrs. 1 lb. is 5⅝ per cent. of what quantity ?
42. 2 qrs. 16 lbs. is 10 per cent. of what quantity ?
43. 1s. 1½d. is 2 per cent. of what sum of money ?
44. 3s. 10¼d. is 5 per cent. of what sum of money ?
45. Which would be more productive to the State, an income tax levied a 3 per cent., or at the rate of 7d. in the £ ? Calculate the tax at th latter rate on an income of £1755, 10s.
46. In Great Britain, in 1849, the population was 18,526,830, and 5¼ pe cent. of the population exercised the elective franchise ; determin the number of electors.
47. £7262, 10s. invested, returns an income of £326, 16s. 3d. ; what rat per cent. is paid on the investment ?
48. If a tax of 12 per cent. on an income of a country brings in £5,200,00 how much will an income tax of 5d. in the pound bring in ? *Answe to the nearest penny.*
49. A broker purchases £15,000 worth of goods. What will be h commission at the rate of ½ per cent. ?
50. What is the cost of insuring a cargo of £52,175, the premium bein 4 per cent. ?
51. What is the premium on an assurance policy of £1895 at 5½ per cent.
52. A tradesman, having promised me 12½ per cent. reduction from a bi of £16, 16s. 8d., takes off only 10 per cent. By what sum am entitled to further diminish the bill ?
53. What commission will a traveller obtain by selling £5760 of goods, h commission being at the rate of ½ per cent. ?

. What percentages (*to the nearest integer*) of the United Kingdom do England, Wales, Scotland, and Ireland form?

Area of England is 32,346,000 acres.
„ Wales „ 4,774,000 „
„ Scotland „ 19,456,000 „
„ Ireland „ 20,334,000 „

. By a certain census, the population of Great Britain was found to be about 18,844,000; the parts employed in agriculture, and trade and manufactures were respectively 1,499,000 and 3,110,000. How much per cent. of the whole population was each of these classes?

. The National Income for the year 1901-2 was £143,581,542, and the main items were :—

Income Tax, ...	£35,378,700
Excise, ...	£31,595,962
Customs, ...	£31,046,869

Express each of these items as a percentage of the whole, *to the nearest whole number.*

. In 1903, the number of German ships that passed through the Suez Canal was 494, while in 1898 the number was 356. Express the increase per cent. as a number *correct to* 1 *place of decimals.*

. In 1903 the American wheat crop consisted of 637,821,000 bushels, and in 1904 increased to 663,000,000 bushels. Find the increase per cent., *correct to* 1 *place of decimals.*

In 1893-4 the Suez Canal shares, held by the British Government, brought in an income of £198,829, while in 1902-3 this sum had increased to £933,778. Express the increase as a percentage *correct to the nearest integer.*

The National Debt increased from £627,563,000 in 1899 to £770,779,000 in 1903. What is the increase per cent? (Correct to the *nearest whole number.*)

61. In 1889-1890 the Navy Estimates were £13,685,400; in 1903-4 they had risen to £34,457,500. Find the increase per cent. *Correct to 1 place of decimals.*

62. In 1889-1890 the Army Estimates were £17,335,800; and in 1903-4 were £34,245,000. Find the increase per cent. *Correct to 1 place of decimals.*

63. Find the percentage error (correct to 2 places of decimals) in writing a^2 for $(a+cx^2)^2$, if $a=3$, $c=1$, $x=0{\cdot}2$.

64. Find the error per cent. (correct to 2 places of decimals) in taking the value of $(\frac{99}{100})^3$ as $1-\frac{3}{100}$.

65. Find the percentage error (correct to 3 places of decimals) in writing $a+bt$ for $a+bt+ct^2$, where $a=1$, $b=3$, $c=2$, $t=\frac{1}{100}$.

66. Given that 1 gram = 15·4323 grains, find the percentage error (correct to 3 places of decimals) in taking it as $15\frac{1}{2}$ grains.

67. Given that 1 metre = 39·3707 inches, find the percentage error (correct to 3 places of decimals) in taking 1 hectometre as $328\frac{1}{3}$ feet.

68. Find the percentage error (correct to 2 places of decimals) if the specific gravity of glycerine is taken as $1\frac{1}{4}$ instead of 1·260.

69. Find the percentage error (correct to 2 places of decimals) in taking the area of the surface of a sphere as $3\frac{1}{7}\times d^2$ instead of $3{\cdot}1416\times d^2$, where d stands for the diameter.

70. Given that 1 kilogram = 2·204 lbs., find the percentage error (to 2 places of decimals) if—

    (i) To convert ozs. to grams, we multiply by $\frac{85}{3}$;

    (ii) To convert lbs. to kilograms, we multiply by $\frac{5}{11}$.

PROFIT AND LOSS

Ex. 1. *If 1 ton of a certain article is bought for £21 and sold at the rate of 2½d. per lb., what is the total gain?*

$$\text{Selling price} = 2\tfrac{1}{2}\text{d.} \times 2240 = £\frac{5 \times 2240}{2 \times 12 \times 20} = £23\tfrac{1}{3} = £23,\ 6\text{s.}\ 8\text{d.}$$

$\therefore$ Total gain = £23, 6s. 8d. − £21 = £2, 6s. 8d. Answer.

N.B.—The gain or loss per cent. is always calculated on the Cost Price.

Ex. 2. *What is the* **gain per cent.** *if an article costing £18 is sold for £22, 10s. 0d.?*

On a cost price of £18 the gain is £4½.

,, ,, £100 ,, ?

$\therefore$ Gain $\% = \frac{9}{2} \times \frac{100}{18} = 25$. Answer.

Ex. 3. *What is the* **selling price** *of an article which costs £15, and is sold at a gain of 5%?*

If the cost price is £100 the selling price is £105.

,, ,, £15 ,, ,, ?

$\therefore$ Selling price $= £105 \times \frac{1}{100} = £\frac{63}{4} = £15,\ 15\text{s.}\ 0\text{d.}$ Answer.

Ex. 4. *If I sell 200 articles for £55 and thus gain 10%, what was the* **cost price** *of each article?*

The selling price is £110 if the cost price is £100.

,, £55 ,, ?

$\therefore$ Cost price of all the articles $= £100 \times \frac{1}{110} = £50$.

$\therefore$ Cost price of 1 article $= £\frac{50}{200} = 5\text{s.}$ Answer.

Exercise LXIII.

1. If 25 tons of goods are purchased for £37, 10s., and sold at 35s. a ton, what is the gain per ton?

2. 1 cwt. of goods are bought at the rate of 1s. 5d. per lb. and sold at £8 the cwt. What is the total gain?

3. A man buys 10 horses at £42 each, and 7 horses at £53 each. If he sells the whole number for £850, what is the total gain?

4. A dealer buys 250 bottles of brandy at 4s. 6d. each; 20 of them are smashed; for how much per bottle must he sell the remainder so as to make 25s. profit altogether?

5. 1 cwt. of sugar is bought for 21s. and sold for 25s. 8d.; what is the gain per pound?

6. A draper buys 171 yards of cloth at 2s. $6\frac{1}{2}$d. per yard. He sells 90 yards at 2s. 9d. per yard, and the rest is accidentally burnt. What is his loss?

7. A merchant buys 5 cwts. of tea for £62. He sells 4 cwts. at the rate of £14 per cwt.; for how much per pound must he sell the rest so as to gain £7, 10s. 8d. on the whole transaction?

8. A man buys a cask of beer, containing 134 gallons, for £11, 18s. Six gallons are lost by leakage. He sells the rest in jugs, each of which holds $\frac{3}{8}$ of a quart, at $2\frac{1}{4}$d. per jug. What did he gain by the bargain?

[*Examples 9 to 20 to be taken orally.*]

What is the gain or loss % if—

9. An article costing £5 is sold for £6.
10. „ „ £20 „ £22.
11. „ „ £40 „ £41.
12. „ „ 30s. „ 33s.
13. „ „ 25s. „ 27s.

4. An article costing 2s. is sold for 2s. 6d.
5. „ „ 2s. 6d. „ 3s.
6. „ „ £50 „ £52, 10s.
7. „ „ £10 „ £10, 10s.
8. „ „ 5s. „ 5s. 6d.
9. „ „ 6d. „ 8d.
10. „ „ £70 „ £75, 10s. ?

Find the **gain or loss per cent.** if—

1. An article costs £7 and is sold for £7, 7s.
2. „ „ £3, 2s. 6d. and is sold for £2, 19s. 4½d.
3. „ „ £7, 10s. 9d. „ £10, 1s.
4. „ „ £18 „ £16.
5. „ „ £20, 10s. 5d. „ £23, 16s. 1d.
6. „ „ £90 „ £83, 14s.
7. „ „ £13 „ £11, 14s.
8. „ costing £1, 15s. per cwt. is sold for 2½d. per lb.
9. „ „ 4s. 2d. per score is sold at the rate of 3s. per doz.
10. Goods purchased at 9s. 4d. per cwt. are sold at 1¼d. per lb.

Find the **selling price** if—

1. An article costing £15 is sold at a gain of 7%.
2. „ „ £6, 5s. „ loss „ 8%.
3. An article costing £8, 10s. 2d. is sold at a gain of 25%.
4. „ „ £41, 13s. 4d. „ „ 8%.
5. „ „ £6, 5s. „ „ 12%.
6. If cheese cost £1, 2s. 11d. per cwt., for what must it be sold per cwt. so as to gain 12 per cent. ?
7. If a merchant purchases sugar for £5, 5s. 10d. and sells it at a loss of 10 per cent., what is the selling price ?
8. A 60-gallon cask of wine was bought for £40, and 20 gallons having been lost by leakage, the remainder was sold so as to gain 5 per cent. on the original outlay. At what price per gallon was it sold ?

What is the **cost price** of an article—

39. Sold at £1, 6s. 4d. at a loss of 20%.
40. „ £2, 3s. 6½d. „ 5%.
41. „ £5, 11s. gain 33⅓%.
42. „ £2, 18s. 6d. „ 4%.
43. „ £1, 10s. 7½d. „ 7⅐%.
44. „ £3, 4s. 2d. loss 12% ?

45. A certain material is sold at £3, 13s. 6d. per dozen yards at a loss of 12½ per cent. ; what was the cost price per yard ?

46. Goods are sold at the rate of 6¾d. per lb. at a gain of 8 per cent. ; what was the cost price per cwt. ?

47. Goods sold for £33 obtain a profit of 10 per cent. ; what was the cost price ?

---

48. A man sells an animal for £35, 2s. at a gain of 17 per cent. ; what did the animal cost him ?

49. By selling an article for 7s. 3d. I gain 16 per cent. on my outlay ; what was its original cost ?

50. A grocer buys tea at £7 per cwt. ; at what price per pound must he sell it so as to gain 20 per cent. ?

51. A man buys goods at £1, 15s. per cwt. ; at what price per qr. must he sell them to gain 10 per cent. on his outlay ?

52. By selling tea at 3s. 1½d. per lb. a grocer loses 25 per cent. ; find the cost price per lb. of the tea.

53. If oranges are bought at the rate of 5 for 2d., what must be the selling price of 60 in order to gain 33⅓ per cent. ?

54. A shopkeeper sold ⅗ of his stock at the price he gave for the whole ; at what rate per cent. is his profit ?

5. If eggs are bought at the rate of 18 for a shilling, how many must be sold for 11s. 3d. so as to gain 25 per cent. ?

6. By selling a house for £570, 10s. a man gains $8\frac{2}{3}$ per cent. ; what did he pay for it ?

7. A sells an article to B at a profit of $8\frac{1}{3}$ per cent., and B sells it to C for £15, 18s. 6d., gaining 40 per cent. What did A give for it ?

8. 360 bottles of wine are bought for £55 and sold at a rate of $39\frac{1}{9}$s. per dozen. What is the gain per cent. ?

9. Coffee passes through the hands of 3 dealers, each of whom makes 5 per cent. profit on his outlay. If it cost the original dealer 1s. 4d. per lb., what will the last one sell it for per cwt. ? (*Answer to the nearest penny.*)

Profit and Loss—*continued*

Ex. 1. *By selling an article for £36 a person gains $33\frac{1}{3}\%$; what would e gain % if he sold it for £33, 15s. ?*

£36 corresponds to £$133\frac{1}{3}$.

£$33\frac{3}{4}$,, ,, ?

$\therefore$ 2nd corresponding selling price $= £133\frac{1}{3} \times \frac{33\frac{3}{4}}{36} = £\frac{400 \times 135}{3 \times 4 \times 36} = £125.$

$\therefore$ The gain % = 25. Answer.

Ex. 2. *By selling an article for £31, 10s. a person gains 5% ; what must the selling price in order that he may lose 4% ?*

£105 corresponds to £$31\frac{1}{2}$.

£96 ,, ,, ?

$\therefore$ Selling price $= £31\frac{1}{2} \times \frac{96}{105} = £\frac{63 \times 96}{2 \times 105} = £\frac{144}{5}$

$= £28, 16s.$

Exercise LXIV.

1. By selling an article for £15, 15s. a person gains 5 per cent.; what would he gain per cent. by selling it for £16, 10s.?

2. By selling an article for £46, 13s. 4d. a person gains $33\frac{1}{3}$ per cent.; what would he gain or lose per cent. by selling it for £33, 12s.?

3. $7\frac{1}{2}$ per cent. is lost by selling an article for £45, 10s.; what would be gained or lost per cent. by selling it for £50, 15s.?

4. If £33 is the selling price of an article when there is a loss of 25 per cent., what will be the gain or loss per cent. by selling it for £52, 16s.?

5. By selling an article for £49, 8s. there is a loss of 5 per cent.; what would be the loss per cent. if it was sold for £48, 15s.?

6. 32 per cent. is gained by selling an article for £34, 13s.; how much will be gained or lost per cent. by selling it for £25, 9s. 3d.?

7. By selling sugar at 3d. per lb. a grocer gains 20 per cent.; at what price per lb. must he sell it so as to lose 30 per cent.?

8. A person sells an article for £33, 19s. 3d. and thus secures a profit of $4\frac{1}{2}$ per cent.; at what price must he sell it if there is a loss of 7 per cent.?

9. Goods are sold at £32, 19s. $0\frac{3}{4}$d. at a gain of $18\frac{3}{4}$ per cent.; for what must they be sold if there is a loss of 10 per cent.?

10. A merchant sells sugar at $3\frac{1}{2}$d. per lb. and thus makes a profit of $16\frac{2}{3}$ per cent.; for how much per cwt. must he sell it so as to obtain a profit of 25 per cent.?

11. By selling tea at 3s. 3d. per lb. a grocer gets a profit of $8\frac{1}{3}$ per cent.; for what price per quarter must he sell it to clear $33\frac{1}{3}$ per cent.?

12. A person loses 17 per cent. by selling an article for £3, 9s. 2d.; at what price should he have sold it so as to gain 20 per cent.?

3. By selling a horse for £82, 5s. a man lost 6⅔ per cent. How much per cent. would he have gained or lost if he had sold it for £90, 10s.?

4. A grocer buys 15 lbs. of tea; he sells 8 lbs. at 2s. 7½d. per lb., and the rest at 2s. 9d. per lb., and finds that he has made 15 per cent. profit. What rate per cent. profit would he have made if he had sold it all at 2s. 9¼d. per lb.?

5. A person bought an estate, and then sold it for £625 less than he gave for it, thereby losing 1¼ per cent.; what should he have sold it for in order to gain 12½ per cent.?

6. A tradesman's prices are 20 per cent. above cost price; if he allows a customer 10 per cent. on his bill, what profit per cent. does he make?

7. When an article is sold at a gain of 5 per cent. it yields 1s. 6d. more than when sold at a loss of 5 per cent. What was the original cost?

8. By selling cloth at 10s. 9½d. per yard, 11 per cent. is gained; at what price must it be sold so as to gain 23 per cent.?

9. A reduction of 20 per cent. in the price of apples would enable a purchaser to obtain 120 more for a sovereign. What may the price of 1 apple be before reduction?

0. A reduction of 10 per cent. in the price of coal would enable a purchaser to obtain for the sum of £13, 10s. two tons more than at the higher price. What may be the price per ton of coal before reduction?

1. Selling articles at 6s. 6d. each, a dealer makes a certain percentage of profit; on increasing the price to 7s. 1d. his percentage of profit is increased by 10. What profit is he making at the latter price?

SIMPLE INTEREST

$$\textbf{Amount} = \textit{Principal} + \textit{Interest}.$$

$$\textbf{Interest} = \frac{\textit{Principal} \times \textit{Time} \times \textit{Rate}}{100}.$$

Time is expressed as a fraction of a year.

If the result is to be *correct to the nearest penny*, then, working in decimals, we must in the final result be *correct to* 3 *decimal places.*

Ex. 1. *Find the simple interest on* £350 *for* 10 *years at* 5%.

$$\text{Interest} = £\frac{350 \times 10 \times 5}{100} = £175.$$

Ex. 2. *Find, correct to the nearest penny, the simple interest on* £535, 18s. 4d. *for* 3 *years at* 4%.

Method i.

$$\text{Interest} = \frac{£535,\ 18s.\ 4d. \times 3 \times 4}{100} = \frac{£535,\ 18s.\ 4d. \times 3}{25}$$

$$= £64,\ 6s.\ 2d.$$

5	£535, 18s. 4d.
5	107, 3s. 8d.
	£21, 8s. 8·8d.
	3
	£64, 6s. 2·4d.

Method ii.

$$\text{Principal} = £535{\cdot}917\ldots.$$

$$\therefore \text{Interest} = \frac{£535{\cdot}917\ldots \times 3 \times 4}{100} = £5{\cdot}35917\ldots \times 12$$

$$= £64{\cdot}310 \text{ (correct to 3 places)}.$$

$$= £64,\ 6s.\ 2d.$$

Ex. 3. *Find, correct to the nearest penny, the simple interest on* £325, 17s. 3d. *for* 187 *days at* 5%.

$$\text{Principal} = £325{\cdot}8625$$

$$\therefore \text{Interest} = £\frac{325{\cdot}8625 \times 187 \times 5}{365 \times 100}$$

$$= £3{\cdot}258625 \times 2{\cdot}56164\ldots$$

$$= £8{\cdot}347 \text{ (correct to 3 places)}$$

$$= £8,\ 6s.\ 11d.$$

3·25863
2·56164
6·51726
1·62932
·19552
·00326
·00195
·00013
8·347

Exercise LXV.

[*Examples* 1–10 *to be answered orally.*]

Find the simple interest on :—

1. £200 for 2 years at 3%.
2. £300 for 5 years at 4%.
3. £500 for 10 years at 12%.
4. £250 for 2 years at 3%.
5. £450 for 4 years at 3%.
6. 1000 francs for 2 years at 7%.
7. 250 francs for 11 years at 4%.
8. 750 dollars for 5 years at 6%.
9. £330 for 20 years at 2% per half-year.
10. £200 for 15 years at $1\frac{1}{2}$% per half-year.
11. £624 for 3 years at 3%.
12. £485 for $2\frac{1}{2}$ years at 3%.
13. £514 for $3\frac{1}{4}$ years at $2\frac{1}{2}$%.
14. £470 for $3\frac{1}{2}$ years at $2\frac{1}{2}$%.
15. £3275 for 3 years at $6\frac{1}{4}$%.
16. £175, 15s. for 2 years at $3\frac{1}{4}$%.
17. £840, 10s. for $3\frac{1}{4}$ years at 7%.
18. £730 for 9 months at $4\frac{1}{2}$%.
19. £630, 10s. for 6 months at $2\frac{1}{2}$%.
20. £450, 12s. 6d. for 4 months at 4%.

Find, *to the nearest penny*, the interest on :—

21. £313 for $2\frac{1}{4}$ years at $2\frac{1}{4}$%.
22. £224 for $1\frac{1}{2}$ years at $5\frac{1}{4}$%.
23. £345, 10s. for $3\frac{1}{2}$ years at 4%.
24. £237, 15s. for $2\frac{1}{4}$ years at 3%.

25. £527, 3s. 4d. for 2 years 10 months at $5\frac{1}{2}\%$.
26. £3050, 10s. 6d. for $3\frac{1}{4}$ years at 5%.
27. £73, 15s. 9d. for 1 year at $3\frac{1}{2}\%$.
28. £207, 12s. 6d. for $21\frac{1}{2}$ years at 3%.
29. £547, 15s. for 5 years at 3%.
30. £127, 9s. $4\frac{1}{2}$d. for 4 years 2 months at $3\frac{1}{8}\%$.
31. £2045, 10s. for 7 years at $3\frac{1}{4}\%$.
32. £41, 13s. 4d. for 8 months at $4\frac{1}{2}\%$.
33. £3168, 15s. for 4 years at $5\frac{1}{2}\%$.
34. £377, 13s. for 1 year 3 months at 5%.
35. £450, 16s. 8d. for 2 years 7 months at $3\frac{1}{4}\%$
36. £720, 5s. for 3 years 5 months at $2\frac{1}{2}\%$.
37. £1585, 17s. 6d. for $6\frac{2}{3}$ years at $4\frac{1}{2}\%$.
38. £315, 15s. for 3 years 2 months at $2\frac{1}{4}\%$.
39. £1505, 2s. $1\frac{1}{2}$d. for $5\frac{1}{2}$ years at 8%.
40. £830, 15s. for $2\frac{1}{4}$ years at $6\frac{1}{4}\%$.
41. £1276, 8s. 4d. for $19\frac{1}{2}$ years at 5%.

Find, *to the nearest penny*, the amount of :—

42. £198, 6s. 8d. for 3 years at $3\frac{1}{2}\%$.
43. £420 for 73 days at $2\frac{1}{2}\%$.
44. £555, 10s. for 219 days at 5%.
45. £2833, 6s. 8d. for $4\frac{1}{2}$ years at 3%.
46. £305, 5s. for 292 days at $6\frac{1}{4}\%$.
47. £135, 6s. 8d. for 146 days at $2\frac{1}{2}\%$.
48. £320 for 225 days at 3%.

INVERSE INTEREST

Ex. 1. *Find what* **sum** *of money will amount to £420 in 4 years at 5%.*

The amount will be £120 if the principal is £100.

,, ,, £420 ,, ,, ?

$\therefore$ Principal $= £100 \times \frac{420}{120} = £350.$

Ex. 2. *In what* **time** *will £720 amount to £828 at 5%?*

Interest $= £828 - £720 = £108.$

The *interest* on £720 is $£\frac{720 \times 5}{100}$ if the time is 1 year.

,, ,, £108 ,, ,, ?

$\therefore$ Time $= 1 \times \frac{100 \times 108}{720 \times 5} = 3$ years.

Ex. 3. *At what* **rate** *per cent. will £550 amount to £660 in 5 years?*

Interest $= £660 - £550 = £110.$

The *interest* on £550 is $£\frac{550 \times 5}{100}$ if the rate is 1%.

,, ,, £110 ,, ,, ?

$\therefore$ Rate% $= 1 \times \frac{100 \times 110}{550 \times 5} = 4.$

N.B.—It should be *very carefully noted* that though

Amount *is* proportional to Principal,

Amount *is not* proportional to Time or Rate.

Exercise LXVI.

1. What sum of money will amount to £504 in 3 years at 4 per cent.?
2. What sum of money lent for 3 years at 5 per cent. will amount to £1058?
3. How much must be lent for 2 years at 5 per cent. to produce £33, 4s. as interest?
4. How much money is lent for $4\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. if the interest on it comes to £112, 12s. 3d.?

5. What sum of money is lent for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent. if the interest amounts to £11, 0s. $9\frac{1}{2}$d. ?
6. What sum of money will amount to £499, 10s. in $5\frac{1}{2}$ years at 2 per cent. ?
7. What sum of money will amount to £474, 19s. 9d. in $4\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. ?
8. What sum of money must be lent for 146 days at $4\frac{3}{4}$ per cent. to produce an interest of £26, 0s. $1\frac{1}{2}$d. ?
9. What sum of money will amount to £769, 5s. in $8\frac{1}{3}$ years at 4 per cent. ?
10. What sum of money amounts to £751, 11s. 3d. in $2\frac{1}{2}$ years at $6\frac{1}{4}$ per cent. ?

In what **time** will—

11. £5174 produce an interest of £258, 14s. at $2\frac{1}{2}$ per cent. ?
12. £550 amount to £632, 10s. at 5 per cent. ?
13. £720 amount to £833, 8s. at $4\frac{1}{2}$ per cent. ?
14. £350 amount to £393, 15s. at 5 per cent. ?
15. £522, 16s. 8d. produce an interest of £39, 4s. 3d. at $2\frac{1}{4}$ per cent. ?
16. £875 produce an interest of £120, 13s. $1\frac{1}{2}$d. at $3\frac{1}{4}$ per cent. ?
17. £3100 amount to £3384, 3s. 4d. at $3\frac{1}{3}$ per cent. ?
18. £1525 produce an interest of £226, 16s. $10\frac{1}{2}$d. at $4\frac{1}{4}$ per cent. ?
19. £11175 produce an interest of £838, 2s. 6d. at 3 per cent. ?
20. £281, 12s. increase to £318, 5s. 4d. at $3\frac{1}{8}$ per cent. ?

At what **rate** per cent. will—

21. £3205 amount to £3653, 14s. in 4 years ?
22. £480 amount to £522 in $3\frac{1}{2}$ years ?
23. £320 produce an interest of £25, 12s. in 2 years ?
24. £240 produce an interest of £24 in 3 years ?
25. £880 amount to £965, 16s. in $3\frac{1}{4}$ years ?
26. £949, 8s. 6d. produce an interest of £197, 15s. $11\frac{1}{4}$d. in $8\frac{1}{3}$ years ?
27. £2400, 16s. 8d. amount to £2995, 0s. $9\frac{1}{2}$d. in $5\frac{1}{2}$ years ?
28. £66, 7s. 1d. amount to £79, 12s. 6d. in 8 years ?
29. £300 increase to £350 in 7 years?
30. £7433, 6s. 8d. amount to £9942, 1s. 8d. in $7\frac{1}{2}$ years ?

Miscellaneous Examples on Simple Interest

Exercise LXVII.

[*Answers are to be given correct to the nearest penny.*]

1. Find the interest on £875, 5s. for 4 years at $5\frac{1}{4}$ per cent.
2. Calculate the interest on £545, 7s. 6d. for $2\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.
3. At what rate per cent. will the interest on £775 for $3\frac{1}{2}$ years amount to £74, 11s. $10\frac{1}{2}$d. ?
4. What is the simple interest on £622, 17s. 6d. for 2 years at $4\frac{1}{2}$ per cent. ?
5. In what time will the interest on £250 at $3\frac{1}{4}$ per cent. amount to £60, 18s. 9d. ?
6. Find the interest on £412, 12s. 6d. for $3\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.
7. What principal, if invested for 3 years at $2\frac{3}{4}$ per cent., will amount to £575, 10s. 7d. ?
8. Find the interest on £253, 13s. 4d. for $2\frac{1}{4}$ years at $3\frac{1}{4}$ per cent.
9. What is the amount of £2674, 15s. for 4 years at $5\frac{1}{2}$ per cent. ?
10. At what rate per cent. will £450 amount to £513 in 4 years ?
11. If £377, 13s. 4d. amounts in a year and a half to £396, 11s., what is the rate of interest ?
12. Find the interest on £420, 10s. for 73 days at 5 per cent.
13. Find the sum of money that will amount to £350, 19s. $4\frac{1}{2}$d. in $6\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.
14. If the interest on £725 for 12 years is £141, 7s. 6d., what is the rate per cent. ?
15. Calculate the interest on £323, 12s. 6d. for $2\frac{3}{4}$ years at $3\frac{1}{2}$ per cent.
16. In what time will £500 amount to £607, 10s. at $5\frac{3}{8}$ per cent. ?

17. What amount of interest does £1020 yield per annum, if invested at $2\frac{3}{4}$ per cent. ?

18. At what rate of simple interest must £230,398, 6s. 8d. be invested in order that it may amount to £298,941, 16s. 9d. at the end of 7 years ?

19. Find the interest on £227, 5s. for 2 years 5 months at $3\frac{1}{4}$ per cent.

20. Find the amount of £435, 10s. in $5\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.

21. At what rate per cent. will a sum of money treble itself in 25 years ?

22. In how many years would £250 double itself at $2\frac{1}{2}$ per cent. ?

23. In what time would a sum of money treble itself at 5 per cent. ?

24. At what rate per cent. would a sum of money quadruple itself in 30 years ?

25. Find the interest on £520, 12s. 6d. from May 2nd to August 31st at 3 per cent.

26. Find the interest on £430, 3s. 4d. from April 1st to September 30th at 4 per cent.

27. Find the interest on £721, 6s. 8d. from April 7th to July 23rd at $3\frac{1}{2}$ per cent.

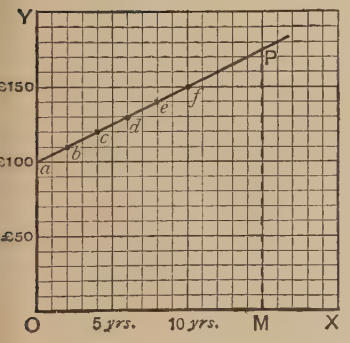
28. Find the interest on £555, 13s. 4d. from February 2nd, 1888, to May 11th at $3\frac{3}{4}$ per cent.

APPLICATIONS OF GRAPHS

Ex. 1. *If the Principal is £100 and the rate per cent. 5, draw a graph showing the connection between Amount and Time, thence find the Amount when the time is 15 years.*

$$\text{Amount} = P + \frac{P \times \text{Rate} \times \text{Time}}{100}$$

$$= 100 + (\text{Time} \times 5).$$



∴ Giving to *Time* the values 0, 2, 4, 6, 8, 10, the corresponding values for *Amount* are 100, 110, 120, 130, 140, 150.

Let distances measured along OX represent Time (1 division represents 1 year).

Let distances measured along OY represent Amount (1 division represents £10).

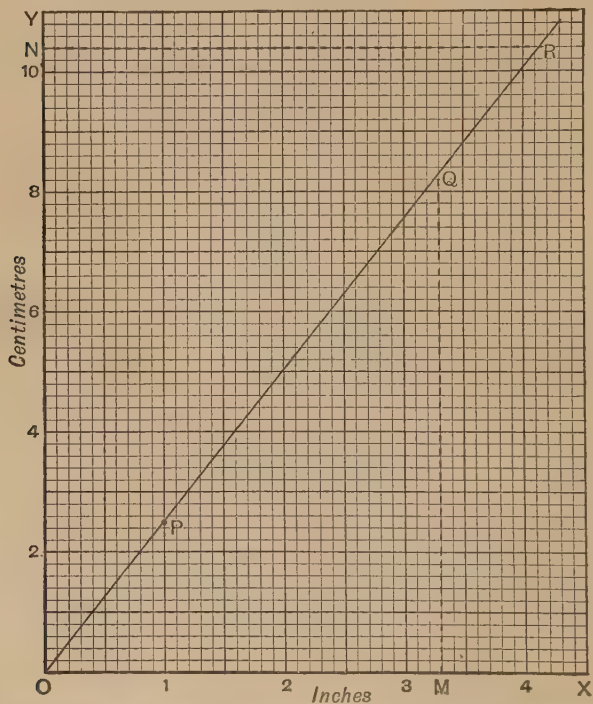
From the above values we obtain the points a, b, c, d, e, f, lying on a straight line *not passing through the origin*,

∴ The Amount *is* **not** *proportional* to the Time.

Take OM to represent 15 years, and erect the perpendicular MP to meet the graph in P, then $MP = 17{\cdot}5$ divisions, and required Amount = £175.

Ex. 2. *Draw a straight line for converting inches into centimetres, given that* 1 *inch* = 2·5 *centimetres. Thence determine the number of centimetres in* 3·3 *inches, and the number of inches in* 10·4 *centimetres.*

1 division along OX is taken to represent ·1 inch.
" " OY " " " ·2 cm.



To obtain one point on the graph (other than the origin), plot the point P, the co-ordinates of which are (1, 2·5) since 1 inch = 2·5 cms. ; joining OP we then have the graph required.

To find the value of 3·3 inches, we take $OM = 3·3$ and then measure off MQ along a line parallel to OY; it is found to be 8·3.

$\therefore$ 3·3 inches = 8·3 cms.

To find the value of 10·4 cms., we take $ON = 10·4$ and then measure NR along a line parallel to OX ; it is found to be 4·15.

$\therefore$ 10·4 cms. = 4·15 inches.

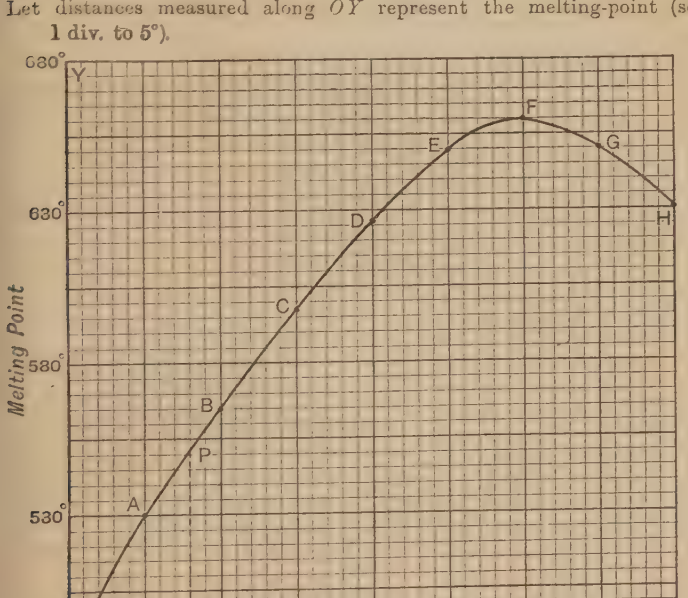
Ex. 3. *The melting-point of a mixture of antimony and copper is shown in the accompanying table :—*

Percentage of copper in mixture.	25	30	35	40	45	50	55	60	65
Melting-point.	480°	530°	565°	598°	627°	650°	660°	650°	630°

Illustrate graphically, and thence determine the melting-point of a mixture containing 33% of copper.

Let distances measured along OX represent the percentage of copper (scale 1 div. to 1%).

Let distances measured along OY represent the melting-point (scale 1 div. to 5°).



From the preceding table the points O, A, B, C, D, E, F, G, H are plotted and joined by a *curved line*, since there are *no sudden breaks* in the

connection between the melting-point and percentage of copper within the limits of temperature chosen.

To determine the melting-point when there is 33% of copper, erect a perpendicular MP, from a point M whose abscissa is 33, to meet the curve in P.

The ordinate of P read off on the scale on OY gives the required melting-point 554°.

Exercise LXVIII.

1. 1, 4, 7, 10, 13, 16 litres are respectively equivalent to 1·8, 7, 12·3, 17·6, 22·9, 28·2 pints. Draw a curve showing the number of pints equivalent to any number of litres between 1 and 16, and thence deduce the value in pints of 3 litres and 12 litres.
2. The price of a standard troy ounce of silver on January 1 in each of the 10 years (1891-1900) was in pence 45, 40, 36, 29, 30, 31, 28, 27, 27, 28. Draw a curve showing its value approximately at any time during these 10 years.
3. The expansions of an iron rod 10 metres long for rises of temperature of 5°, 10°, 15°, 20°, 25° are respectively ·6, 1·2, 1·8, 2·3, 2·9 millimetres. Draw a graph illustrating this, and thence deduce the expansions for rises of 4°, 11° and 17°.
4. The range of a certain gun is 174, 342, 500, 643, 766, 866, 940 yards, when the elevation of the gun is 5, 10, 15, 20, 25, 30, 35 degrees respectively. Draw a curve showing how the range varies from 5° to 35°, and thence find the range when the elevation is 12°.
5. The temperature, in a certain experiment, taken at the end of 5, 10, 15, 20, 25 seconds, was found to be 10·5, 14·3, 16, 14·4, 6 degrees respectively. Draw a curve showing the change in temperature, and deduce the approximate value at the end of 7, 17, and 23 seconds.
6. If a very thin rod, 9 inches long, swings like a pendulum, then the tendency of the rod to break at distances 0, 1, 3, 4, 5, 7, 9 inches from the point of suspension is 0, 64, 108, 100, 80, 28, 0 units respectively. Illustrate these results by a graph, and then obtain the tendency to break at distances 2, 6, and 8 inches from the point of suspension.

7. A boat, on leaving port, gradually gets up steam, and at the end of 15, 25, 35, 50, 75, 100, 110, 125, 130 minutes respectively, is steaming 1, 4, 7, 9, 10, 12, 15, 17, 17 knots. Draw a graph showing how the speed varies, and from it determine approximately the number of knots she is making at the end of 30, 90, and 115 minutes.

8. The electrical resistance of pure copper at 0° being taken as 1, the resistance at temperatures of 2, 6, 9, 11, 13, 14 degrees respectively are 1·008, 1·023, 1·034, 1·042, 1·050, 1·054. From the graph which illustrates the change in resistance, find approximately the resistances at temperatures of 4, 7, and 12 degrees.

9. The National Expenditure for consecutive financial years is shown in the following table :—

Year.	1893	'94	'95	'96	'97	'98	'99	1900	'01	'02
Expenditure in millions of pounds.	78	79	82	86	90	91	96	119	168	180

Plot a curve showing how the expenditure varied.

0. Draw a graph illustrating the variation in the population of England and Wales since 1801.

Year.	1801	'11	'21	'31	'41	'51	'61	'71	'81	'91	1901
Population in millions.	8·9	10·2	12	14·2	15·9	17·9	20·1	22·7	26	29	32·5

Thence determine the approximate populations in 1847 and 1879.

1. Draw a similar curve for Ireland.

Year.	1821	'31	'41	'51	'61	'71	'81	'91	1901
Population in millions.	6·8	7·8	8·2	6·6	5·8	5·4	5·2	4·7	4·5

Estimate the approximate populations in 1847 and 1867.

12. In a steam vessel of 8000 tons, it is found that the horse-power required to drive the vessel at 5, 10, 15, 20 knots respectively is 228, 1827, 6164, and 14612. Illustrate this by a graph, and from it determine the approximate horse-power required to propel the ship at 12 knots.

13. The relation between the electric power in kilowatts sent out by an electric-lighting station and the amount of coal burnt in the boiler furnaces per hour is—

Power in kilowatts.	1	3	6	12	15
Number of pounds of coal burnt per hour.	541·7	545	550	560	565

Thence determine the number of lbs. of coal used to generate 8 kilowatts.

14. The 'Expectation of Life' for males and females at various ages is shown in the following table. Exhibit this by a graph, and find the expectation of life (approximately) of a man 23 years, and a woman 42 years old.

Age.	5	10	15	20	25	30	35	40	45
Expectation of males in years.	50·9	47·6	43·4	39·4	35·7	32·1	28·6	25·3	22·1
Expectation of females.	53·1	49·8	45·6	41·7	38	34·4	30·9	27·5	24·1

15. The corresponding readings on a Fahrenheit and a Centigrade thermometer are shown. Illustrate by a graph, and from it calculate the Fahrenheit readings corresponding to 44° C. and 62° C.

Fahrenheit reading.	104	113	122	140	158
Centigrade reading.	40	45	50	60	70

16. The connection between the total cost of steam plant of a certain kind and the maximum indicated horse-power is—

Indicated horse-power.	5	10	12	18	21
Total cost in £.	200	300	340	460	520

Exhibit by a graph, and thence find the approximate cost of steam plant yielding 15 horse-power.

17. The solubility of Mercuric Chloride in water at various temperatures is shown in the following table :—

No. of gms. dissolved in 100 gms. of water.	6	7	9·7	14	24·5	55
Centigrade temperature.	0°	20°	40°	60°	80°	100°

Estimate the probable solubilities at 35° and 90°.

18. Plot a curve showing the variation in the solubility of Strontium Nitrate—

No. of gms. dissolved in 100 gms. of water.	30	71	87	94	95	98·5
Centigrade temperature.	0°	20°	30°	50°	70°	90°

Estimate the probable solubilities at 10° and 38°.

19. The area of the segment of a circle is $\frac{h^3}{2c}+\frac{2}{3}ch$, where c is the length of the chord, and h the height of the segment. Find the area in sq. metres (correct to 1 place of decimals) when $c=5$ metres and $h=1, 2, 5, 7$ metres respectively.

Draw a graph and determine the area of the segment when $h=6{\cdot}5$ metres, and find the value of h in order that the area may be 12 sq. metres.

20. The volume of a solid circular ring is $2\pi^2 Rr^2$, where r is the radius of the cross section, R is the mean radius of the ring, and $\pi = 3{\cdot}14$.

Find the volume when $R = 5$ cms. and $r = 1, 3, 5, 6$ cms. respectively. Draw a graph showing the connection between the volume and r; thence read off the volume when $r = 3{\cdot}3$ and the radius when the volume is 360 cu. cms.

21. Construct a graph from which may be read off the cube roots of numbers between 8 and 27; thence find the cube roots (correct to 2 places) of 12·4 and 14·8, and the approximate cubes of 2·81 and 2·95.

22. Plot $y = x^2$, by giving to x the values of 9 and 9·1, thence find the square roots of 81·45 and 8240.

23. The charge (in pounds) of untamped gun cotton required to demolish a brick pier of length L feet and thickness T feet is $\frac{2}{3}LT^2$. If $L = 220$, tabulate the values (*to the nearest pound*) of the charge for values 1, 2, 3, 4, 5 feet thickness respectively.

Illustrate graphically, and thence determine the charge when $T = 2{\cdot}4$.

COMPOUND INTERESTS

Ex. 1. *Find, to the nearest penny, the amount, at compound interest, of* £1500, 6s. 8d. *for* $2\frac{1}{2}$ *years at* $3\frac{1}{2}\%$.

	£1500·333 33	= Principal for 1st year.
$1\frac{3}{100}$	45·009 99	= Interest ,, ,,
$2\frac{1}{100}$	7·501 66	
	1552·844 98	= Principal ,, 2nd ,,
$1\frac{3}{100}$	46·585 34	= Interest ,, ,,
$2\frac{1}{100}$	7·764 22	
	1607·194 54	= Principal ,, 3rd ,,
$1\frac{1}{100}$	16·071 94	
$2\frac{1}{100}$	8·035 97	= Interest for 1st half of 3rd year.
$4\frac{1}{100}$	4·017 98	
	£1635·320.	

∴ Amount = £1635, 6s. 5d.

$$\left[3\tfrac{1}{2}\% = \frac{3\frac{1}{2}}{100} = \frac{3}{100} + \frac{1}{200}\right.$$

$$\left. 1\tfrac{3}{4}\% = \frac{1+\frac{1}{2}+\frac{1}{4}}{100} = \frac{1}{100} + \frac{1}{200} + \frac{1}{400}\right]$$

Ex. 2. *Find the Principal which in* 2 *years at* 5% *amounts to* £3450, 6s. 8d.

Suppose the Principal is £100.

	£100	= Prin. for 1st year.
$1\frac{5}{100}$	5	= Int. ,, ,,
	105	= Prin. ,, 2nd ,,
$1\frac{5}{100}$	5·25	= Int. ,, ,,
	£110·25	= Amount.

∴ Amount is £110·25 if Principal is £100.

,, £3450·$\dot{3}$,, ,, ?

∴ Principal = £$\frac{345033\cdot\dot{3}}{110\cdot25}$.

= £3129·554.

= £3129, 11s. 1d.

	3129·5540
1·1025	3450·3333
	3307 5
	142 83
	110 25
	32 583
	22 050
	10 5333
	9 9225
	6108
	5513
	595
	551
	44
	44

When the time is long, this method obviously becomes very cumbersome; in such cases, the problems may be solved either by using Compound Interest Tables (a specimen is given below) or from Seven-Figure Logarithms by means of the formula $M = PR^n$, where M is the amount, P the principal, R the amount of £1 for 1 year and n the number of years.

Table giving the Amount of £1 at Compound Interest

Yr.	2½%	3%	3½%	4%	4½%	5%
1	1·0250	1·0300	1·0350	1·0400	1·0450	1·0500
2	1·0506	1·0609	1·0712	1·0816	1·0920	1·1025
3	1·0769	1·0927	1·1087	1·1249	1·1412	1·1576
4	1·1038	1·1256	1·1475	1·1699	1·1925	1·2155
5	1·1314	1·1593	1·1877	1·2167	1·2462	1·2763
6	1·1597	1·1941	1·2293	1·2653	1·3023	1·3401
7	1·1887	1·2299	1·2723	1·3159	1·3609	1·4071
8	1·2184	1·2668	1·3168	1·3686	1·4221	1·4775
9	1·2489	1·3048	1·3629	1·4233	1·4861	1·5513
10	1·2801	1·3439	1·4106	1·4802	1·5530	1·6289
11	1·3121	1·3842	1·4510	1·5395	1·6229	1·7103
12	1·3449	1·4258	1·5111	1·6010	1·6959	1·7959
13	1·3785	1·4685	1·5640	1·6651	1·7722	1·8856
14	1·4130	1·5126	1·6187	1·7317	1·8519	1·9799
15	1·4483	1·5580	1·6753	1·8009	1·9353	2·0789
16	1·4845	1·6047	1·7340	1·8730	2·0224	2·1829
17	1·5216	1·6528	1·7947	1·9479	2·1134	2·2920
18	1·5597	1·7024	1·8575	2·0258	2·2085	2·4066
19	1·5987	1·7535	1·9225	2·1068	2·3079	2·5270
20	1·6386	1·8061	1·9898	2·1911	2·4117	2·6533
21	1·6796	1·8603	2·0594	2·2788	2·5202	2·7860
22	1·7216	1·9161	2·1315	2·3699	2·6337	2·9253
23	1·7646	1·9736	2·2061	2·4647	2·7522	3·0715
24	1·8087	2·0328	2·2833	2·5633	2·8760	3·2251
25	1·8539	2·0938	2·3632	2·6658	3·0054	3·3864

Ex. 3. *Given that the amount of £1 for 20 years at 3% is £1·8061, calculate the amount of £243, 8s. 7d. for the same time and rate.*

£243, 8s. 7d. = £243·42917 (correct to 5 places).

∴ Amount = £243·42917 × 1·8061.

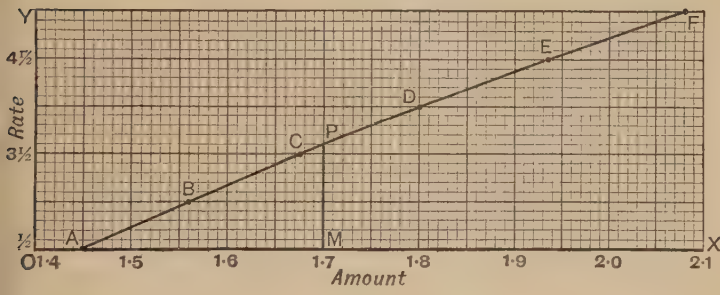
```
243·42917
  1·8061
---------
243·42917
194·74334
 1·46057
  ·02434
---------
```

∴ Amount = £439·657.

= £439, 13s. 2d. (approx.).

Ex. 4. *Given the following table showing the connection between the Amount of £1 at the end of 15 years and the rate per cent., draw a curve and thence determine approximately the rate at which the amount is £1, 14s. 0d.*

Rate %.	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5
Amount of £1.	1·448	1·558	1·675	1·801	1·935	2·079



Let the Amounts be measured along OX ; 1 division represents £0·01 ;

,, Rates ,, ,, ,, OY ; 10 divisions represent 1%.

From the above table the points A, B, C, D, E, F are plotted and the curve drawn.

To determine the rate at which the amount is £1, 14s. 0d. or £1·7, take M, the abscissa of which represents an amount of £1·7 and erect a perpendicular to meet the curve in P, then the rate is represented by the ordinate of P and is 3·6 or $3\frac{3}{5}$ (approx.).

Exercise LXIX.

[*Answers to be given to the nearest penny.*]

Find the *amount* at compound interest of–

1. £500 for 2 years at 5 per cent.
2. £550 for 2 years at 4 per cent.
3. £480 for 2 years at 3 per cent.
4. £530 for 3 years at 4 per cent.
5. £750 for 3 years at 3 per cent.
6. £626 for 3 years at 5 per cent.
7. £827 for 3 years at 6 per cent.
8. £742 for 3 years at 7 per cent.

9. £495 for 2 years at $2\frac{1}{2}$ per cent.
10. £524, 10s. for 2 years at $3\frac{1}{4}$ per cent.
11. £635, 5s. for 2 years at $4\frac{3}{4}$ per cent.
12. £4500 for 2 years at $4\frac{1}{4}$ per cent.
13. £133, 6s. 8d. for 3 years at $2\frac{1}{2}$ per cent.
14. £2342, 11s. 4d. for 4 years at $2\frac{1}{2}$ per cent.
15. £4765 for 3 years at $3\frac{3}{4}$ per cent.
16. £3689, 8s. 3d. for $2\frac{1}{3}$ years at $4\frac{1}{2}$ per cent.
17. £4250 for $2\frac{1}{2}$ years at 5 per cent.
18. £104 for $1\frac{3}{4}$ years at 5 per cent.
19. £7123, 4s. 7d. for $2\frac{1}{4}$ years at 4 per cent.
20. £205 in $2\frac{1}{2}$ years at $7\frac{1}{4}$ per cent.
21. £540 for 2 years at 4 per cent. per annum, payable half-yearly.
22. £1025, 10s. for $1\frac{1}{2}$ years at 6 per cent. per annum, payable half-yearly.
23. £1125, 5s. for $1\frac{1}{2}$ years at 3 per cent. per annum, payable half-yearly.
24. £750, 17s. 6d. for $1\frac{1}{2}$ years at 6 per cent. per annum, payable quarterly.

Find the *sum of money* which at compound interest amounts to—

25. £2400 in 2 years at 5 per cent.
26. £1050 in 3 years at 4 per cent.
27. £1764 in 3 years at 6 per cent.
28. £1500 in 3 years at 7 per cent.
29. £413, 8s. 9d. in 2 years at 5 per cent.
30. £1352 in 2 years at 4 per cent.
31. £463, 1s. in 3 years at 5 per cent.

Find the *sum of money*, the compound interest on which amounts to—

32. £423, 17s. 7d. in 3 years at 8 per cent.
33. £122, 11s. 5d. in 3 years at $3\frac{1}{2}$ per cent.
34. £144, 11s. 9d. in 3 years at $2\frac{1}{2}$ per cent.

From the Compound Interest Tables, calculate approximately the Amount of—

35. £325, 5s. 0d. for 17 years at $3\frac{1}{2}$ per cent.

36. £1400, 8s. 0d. for 13 years at 4 per cent.

37. £5782, 2s. 3d. for 21 years at $4\frac{1}{2}$ per cent.

38. £437, 15s. 4d. for 19 years at 3 per cent.

39. £1572, 14s. 5d. for 16 years at $3\frac{1}{2}$ per cent.

40. Draw a curve representing the connection between the time and amount of £1 at 3 per cent. for 1, 2, 3 . . . 15 years respectively: thence determine approximately the time for which the amount is £1, 5s. 1d.

STOCKS AND SHARES

Ex. 1. How much 4% **stock** *at* 105 *can be bought for* £2000 ?

£105 will purchase £100 stock.
£2000 ,, ?
∴ Amount of stock = £100 × $\frac{2000}{105}$ = £$\frac{40000}{21}$.
= £1904·762.
= £1904, 15s. 3d.

3|40000
7|13333·3333
1904·7619

Ex. 2. *Find the* **cost of** £3000 *of* 5% *stock at* 110.

£100 stock may be purchased for £110.
£3000 ,, ,, ?
∴ Cost = £110 × $\frac{3000}{100}$ = £3300.

Ex. 3. *What is the annual* **income** *derived from* £5000, 17s. 6d. *of* 5% *stock?*

£100 stock yields an income of £5.
£5000·875 ,, ,, ?
∴ Income = £5 × 50·00875 = £250·044.
= £250, 0s. 11d.

Ex. 4. *What* **income** *is derived from investing* £3500 *in* 4% *stock at* 101 ?

An investment of £101 yields an income of £4.
,, £3500 ,, ?
∴ Income = £4 × $\frac{3500}{101}$.
= £138·614.
= £138, 12s. 3d.

138·6139
1·01)140·0000
101
390
303
870
808
620
606
140
101
390
303
9
9

Ex. 5. What is the price *of stock, if an income of* £36 *is obtained by investing* £1164 *in the* 3 *per cents.?*

£36 income is yielded by an investment of £1164.

£3 ,, ,, ,, ?

$\therefore$ Price of stock $= £1164 \times \frac{3}{36} = 97.$

Ex. 6. How much money *will be obtained by selling* £5000 *stock at* $95\frac{3}{8}$, *brokerage being at* $\frac{1}{2}$ *per cent.?*

£100 stock will sell for £$94\frac{7}{8}$.

£5000 ,, ?

$\therefore$ Money obtained $= £94{\cdot}875 \times 50.$

$= £4743{\cdot}75 = £4743,\ 15s.\ 0d.$

Ex. 7. £2000 *is invested in the* 3 *per cents. at* 90, *sold at* 92, *and re-invested in the* 4 *per cents. at* 115. *What is the* **change in income?**

£90 invested in the 3 per cents. yields income of £3.

£2000 ,, ,, ?

$\therefore$ 1st Income $= £3 \times \frac{2000}{90} = £66,\ 13s.\ 4d.$

Since the selling price is 92 while the buying price is 90, the amount of money obtained by sale of stock is £$2000 \times \frac{92}{90}$.

Now £115 invested in 4 per cents. yields income of £4.

£$\frac{2000 \times 92}{90}$,, ,, ?

$\therefore$ 2nd Income $= £4 \times \frac{2000 \times 92}{115 \times 90} = £\frac{640}{9}.$

$= £71{\cdot}111 = £71,\ 2s.\ 3d.$

$\therefore$ Change of income $= £4,\ 8s.\ 11d.$

Exercise LXX.

[*Answers to be given correct to the nearest penny.*]

How much stock can be obtained by investing—

1. £428 in New South Wales 4 per cent. stock at 107?
2. £434 „ New Zealand 4 „ „ „ 108½?
3. £978, 10s. in Queensland 4 per cent. stock at 103?
4. £862, 15s. „ Victoria 4 „ „ „ 101⅜? (Brokerage ⅛ pe cent.)
5. £942, 15s. in Grand Trunk Railway Guaranteed 4 per cent. stock a 104¾?

What **amount** of—

6. Argentine 5 per cent. stock at 91 must be sold to realise £864, 10s.?
7. Austrian 4 per cent. Gold Rentes at 102 must be sold to realis £841, 10s.?
8. Brazilian 4½ per cent. stock at 83 must be sold to realise £809, 5s.?
9. Bulgarian 6 per cent. stock at 92⅜ must be sold to realise £500 (Brokerage ⅛ per cent.)
10. Chinese 7 per cent. stock at 89⅝ must be sold to realise £250 (Brokerage ⅛ per cent.)

Find **the cost** of—

11. £900 of Chinese 6 per cent. stock at 103.
12. £800 of Colombian stock at 31.
13. £1500 of Costa Rica stock at 23¾. (Brokerage ⅛ per cent.)
14. £1200 Egyptian 3½ per cent. Preference stock at 3 above par.
15. 155 £1 shares in a company at 1⅜.
16. 250 £2 „ „ „ 2⅝.

How much will be received **from the sale** of—

17. £5000 Italian 5 per cent. stock at 101⅝? (Brokerage ⅛ per cent.)
18. £4500 Japanese 5 per cent. stock at 104½? (Brokerage 2s. 6d. per cent
19. £5700 Mexican 5 per cent. Consolidated stock at 101½?
20. £4200 Norwegian 3 per cent. stock at 92⅛? (Brokerage ⅛ per cent.)
21. £6500 Portuguese 3 per cent. stock at 63⅜?

What **annual income** is obtained from—

22. £2000 Swedish $3\frac{1}{2}$ per cent. stock?
23. £2500 Turkish 4 per cent. stock?
24. £1520 Turkish $3\frac{1}{2}$ per cent. stock?
25. £850 Queensland $3\frac{1}{2}$ per cent. stock?
26. £1120, 5s. New Zealand 4 per cent. stock?
27. £5700 stock paying dividends at the rate of 4 per cent. per annum for the first half-year, and $3\frac{1}{4}$ per cent. per annum for the second half-year?
28. 875 £1 shares in a company paying a quarterly dividend of 7d. per share?
29. 527 £5 shares in a company paying half-yearly dividends of 8s. per share?
30. 625 £5 shares in a company paying a half-yearly dividend of 7s. 6d. per share?
31. The possession of £875 stock paying half-yearly dividends at the rate of 3 per cent. per annum for the first half-year, and $1\frac{1}{4}$ per cent. per annum for the second half-year?

What **yearly dividend** is obtained **from investing**—

32. £1200 in the 3 per cent. stocks at 97?
33. £1500, 10s. in $2\frac{1}{2}$ per cent. Consols at $90\frac{3}{8}$? (Brokerage $\frac{1}{8}$ per cent.)
34. £1700, 17s. 6d. in Exchequer $2\frac{3}{4}$ per cent. Bonds at $99\frac{5}{8}$?
35. £2500, 6s. 8d. in India $3\frac{1}{2}$ per cent. stock at $105\frac{1}{2}$?
36. £5950 in India $2\frac{1}{2}$ per cent. stock at $84\frac{7}{8}$? (Brokerage 2s. 6d. per cent.)
37. £7582, 10s. in London County $2\frac{1}{2}$ per cent. stock at $85\frac{1}{2}$?
38. £9000 in London County 3 per cent. stock at $96\frac{1}{4}$?
39. £2550 in £10 shares at $5\frac{1}{2}$, the company paying dividends at 15 per cent.?
40. £275 in £1 colliery shares at $3\frac{1}{4}$, the company paying dividends at 18 per cent.?

What is **the price** of a—

41. 3 per cent. stock, if by investing £1550 an income of £50 is produced

42. $2\frac{3}{4}$ per cent., if by investing £1665 an income of £49, 10s. is produced (Brokerage $\frac{1}{8}$ per cent.)

43. $1\frac{1}{2}$ per cent. stock, if by investing £3289 an income of £156 is produced?

44. $1\frac{1}{4}$ per cent. stock, if by investing £1057, 10s. an income of £58, 15s is produced? (Brokerage $\frac{1}{8}$ per cent.)

45. $3\frac{1}{4}$ per cent. stock, if by investing £6844, 10s. an income of £234 is produced?

46. 3 per cent. stock, if by investing £351, 0s. 6d. an income of £11, 8s. is produced?

What **rate of interest** is obtained by investing in—

47. 5 per cent. stock at 75?
48. 4 „ „ 84?
49. $3\frac{1}{2}$ „ „ 63?
50. $2\frac{1}{2}$ „ „ 35?
51. $4\frac{3}{4}$ „ „ 76?
52. $6\frac{1}{2}$ per cent. stock at 78?
53. $5\frac{1}{4}$ „ „ 105?
54. $3\frac{1}{3}$ „ „ 75?
55. 7 „ „ 91?
56. $6\frac{1}{4}$ „ „ 95?

57. If the 4 per cent. stocks pay interest at the rate of $5\frac{1}{3}$ per cent. on the money invested, what is the *price of the stock*?

58. If the $3\frac{1}{2}$ per cent. stocks pay interest at the rate of $4\frac{1}{6}$ per cent. on the money invested, what is the price of the stock?

59. If the 7 per cent. stocks pay interest at the rate of $11\frac{1}{9}$ per cent. on the investment money, what is the price of the stock?

60. If the 5 per cent. stocks pay interest at the rate of 4 per cent. on the investment money, what is the price of the stock?

61. If the $2\frac{1}{4}$ per cent. stocks pay interest at the rate of $2\frac{7}{9}$ per cent. on the investment money, what is the price of the stock?

Which is the **better investment**, the—

32. 3 per cents. at 72, or 4 per cents. at 80?
33. $2\frac{1}{2}$ „ „ 75, or $4\frac{1}{2}$ „ „ 140?
34. $4\frac{1}{4}$ „ „ 84, or $5\frac{1}{2}$ „ „ 92?
35. $3\frac{1}{2}$ „ „ 93, or 4 „ „ 97?
36. $2\frac{3}{4}$ „ „ $77\frac{1}{2}$, or $3\frac{1}{2}$ „ „ $85\frac{1}{2}$?
37. $3\frac{1}{4}$ „ „ $91\frac{1}{2}$, or $3\frac{1}{2}$ „ „ 94?

Find the **change in income** when—

38. £5000 of 3 per cent. stock at 95 is sold, and the money obtained invested in a 4 per cent. stock at 102.

39. £7500 of $2\frac{1}{2}$ per cent. stock at 92 is sold, and the money obtained invested in a 3 per cent. stock at 99.

40. £6400 of a $2\frac{1}{4}$ per cent. stock at 88 is sold, and the money obtained invested in a $3\frac{1}{2}$ per cent. stock at 105.

41. The money obtained from the sale of £5600 of $3\frac{1}{2}$ per cent. stock at 97 is invested in a $3\frac{1}{4}$ per cent. stock at 96.

42. The money obtained from the sale of £7300 of 4 per cent. stock at 105 is invested in a $5\frac{1}{2}$ per cent. stock at 110.

43. A person invests £2000 in a 3 per cent. stock at 95, and when the stock has risen to 96 he sells out and invests the money obtained in a 4 per cent. stock at 102. What is his change in income?

Miscellaneous Examples on Stocks

Exercise LXXI.

[*Answers are to be given correct to the nearest penny.*]

1. Find the income derived from investing £3000 in $2\frac{1}{2}$ per cent Consols at $90\frac{3}{8}$. (Brokerage $\frac{1}{8}$ per cent.)

2. How much will be obtained by selling £2600 of Portuguese 3 per cent. stock at $63\frac{3}{8}$? (Brokerage $\frac{1}{8}$ per cent.)

3. How much B. Columbia 3 per cent. stock at 89 can be bought for £1500?

4. Find the price of 3 per cent. stock when an investment of £434, 12s. 6d produces an income of £14, 5s.

5. A person investing in the 3 per cents. receives $3\frac{1}{3}$ per cent. interest on his money. What is the price of the stock?

6. A man invests £1274 in the $3\frac{1}{2}$ per cents. at 91. What is his income

7. If the 3 per cent. stock be at 98, and the $3\frac{1}{4}$ per cent. stock at 101 which stock is it most advantageous to buy? What income will £5000 invested in the 3 per cent. stock produce?

8. A man bought £500 of 3 per cent. Consols at $93\frac{1}{2}$, and after having received one dividend, he sold at $96\frac{3}{4}$. What did he gain by the transaction?

9. If the $3\frac{1}{2}$ per cents. be at $95\frac{5}{8}$ and the 3 per cents. at 82, which is the better investment?

10. A person having invested a sum of money in the 3 per cent. Consols receives annually £233 after deducting the income tax of 7d. in the £; what is the amount of stock he holds? What can the stock be sold for when the Consols are at $94\frac{1}{8}$?

11. A person bought into the 3 per cents. at 98, and after receiving 3 years' interest, he sold at 90? How much per cent. on the sum invested did he gain or lose?

12. If I invest £3591 in 3 per cents. at $85\frac{1}{2}$, what income shall I get, and what interest per cent. on the investment?

13. If I invest £3370 in a stock when it is at $84\frac{1}{8}$, how much of it shall I purchase? (Brokerage $\frac{1}{8}$ per cent.)

14. Which is the better investment, the 4 per cents. at 120 or the $2\frac{1}{2}$ per cents. at 75?

15. What is the gain by investing £1950 at $97\frac{3}{8}$, and selling out at 104? (Brokerage $\frac{1}{8}$ per cent.)

16. How should I affect my income by selling out from the $1\frac{3}{4}$ per cents. at $47\frac{1}{4}$, and investing in the $3\frac{2}{3}$ per cents. at 99?

17. A man invests £5000 in the $3\frac{1}{2}$ per cents. when they are at $73\frac{1}{2}$. What is his income?

18. What sum must be invested in 4 per cent. Spanish stock at $63\frac{1}{2}$ to give an income of £340, 4s. 8d.?

19. Would it be more advantageous to invest in the $3\frac{1}{2}$ per cents. at $80\frac{5}{8}$, or in the 4 per cents. at $93\frac{1}{2}$?

20. What annual income is gained by investing £1900 in the $3\frac{1}{2}$ per cents. at 95? Find the change in income if the stock be sold at 90, and the proceeds reinvested in the 3 per cents. at 81.

21. A person has a sum invested in the 3 per cent. stock, which he sells, and invests in the $3\frac{1}{2}$ per cents. at $87\frac{1}{2}$. If his income remain the same, what is the price of the 3 per cent. stock?

22. A person having £1000 invests in the 3 per cents. at 92, and pays a broker $\frac{1}{2}$ per cent. on the stock purchased. After 3 years he sells at 95, and again pays a broker $\frac{1}{2}$ per cent. What did he receive as interest, and what did he gain by the sale?

23. A person sells out 4 per cent. stock at 126, and invests the proceeds in equal parts in $2\frac{3}{4}$ per cent. stock at 98 and 5 per cent. stock at 120. How much per cent. does he increase his income by the change? (The answer is to be correct to 2 places of decimals.)

24. A railway stock pays a dividend of $6\frac{3}{4}$ per cent. ; what price should a person pay for £100 of the stock that he may have 4 per cent interest on his money ?

25. A man invests one-third of his capital in $2\frac{1}{2}$ per cent. stock at 96, one quarter in 4 per cent. stock at 110, and the remainder in 5 per cent stock at 120 ; find, correct to the nearest penny, the average rate per cent. returned on the capital.

26. A man invests equal sums in 3 per cent. stock at 96 and in 4 per cent Railway Debentures at 127. What percentage did he get for his money on the whole ? (Answer to the nearest penny.)

27. There are three speculations which bring in respectively 5, 7, and 9 per cent. on the money invested. Compare the average interest on their capitals obtained by two speculators, one of whom invests an equal sum in each, and the other invests such sums that he obtains an equal income from each.

28. A person derives an income of £253, 10s. from an investment in the 3 per cent. Consols. He sells out when they are at $93\frac{7}{8}$, and invests the proceeds in Brazilian 5 per cents. at $97\frac{1}{2}$. Find the increase of his income.

PRESENT WORTH AND DISCOUNT

Present Worth corresponds to *Principal* in problems on Interest.
Amount of Bill ,, *Amount* ,, ,,
True Discount ,, *Interest on Present Worth.*
Banker's Discount ,, *Interest on Amount of Bill.*

Present Worth + True Discount = Amount of Bill.

∴ True Discount + Interest on True Discount = Interest on Bill.
= Banker's Discount.

∴ *Banker's Discount – True Discount = Interest on True Discount.*

Ex. 1. *Find the Present Worth and True Discount on a bill of £293, 15s. due $3\frac{1}{2}$ years hence at 5%.*

[This is the same as finding the Principal and Interest when the Amount is given.]

The amount of the bill is £$117\frac{1}{2}$ if the P. W. is £100
,, £$293\frac{3}{4}$,, ?

$$\therefore \text{P. W.} = £100 \times \frac{293\frac{3}{4}}{117\frac{1}{2}} = £\frac{100 \times 1175 \times 2}{235 \times 4}$$

$$= £250$$

∴ Discount = £293, 15s. – £250 = £43, 15s.

Ex. 2. *If the true* **discount** *on a bill due 2 years hence at 3% is £35, what is the* **amount** *of the bill?*

[This corresponds to finding the Principal, thence the Amount, given the Interest.]

The discount is £6 if the P. W. is £100
,, £35 ,, ,,

$$\therefore \text{P. W.} = £100 \times \tfrac{35}{6} = £583, 6s. 8d.$$

∴ Amount of bill = £618, 6s. 8d.

Ex. 3. When is the bill due *if the Present Worth of £134, 8s. 0d. at 5% is £120?*

[This corresponds to finding the time, when the Principal, Amount, and Rate are given.]

Discount = £$134\frac{2}{5}$ – £120 = £$14\frac{2}{5}$.

n a Present Worth of £120 the discount is $\left(\frac{120 \times 3}{100} = \right)$ £$\frac{18}{5}$ if the time is 1 year.

. ,, ,, ,, ,, £$14\frac{2}{5}$,, ,, ?

$$\therefore \text{Time} = \frac{14\frac{2}{5}}{\frac{18}{5}} = \frac{72 \times 5}{5 \times 18} = 4 \text{ years.}$$

Exercise LXXII.

[*The answers are to be given correct to the nearest penny.*]

What is the **amount of a bill** due :—

1. 2 years hence, if its present worth is £200 and the rate per cent. 5 ?
2. 4 „ „ „ „ „ £350 „ „ 5 ?
3. 4 „ „ „ „ „ £475 „ „ $3\frac{1}{3}$?
4. $3\frac{1}{2}$ „ „ „ „ „ £675, 15s. „ „ $5\frac{1}{2}$?
5. 8 months „ „ „ „ £1550 „ „ $3\frac{1}{3}$?
6. 2 years 4 months „ „ „ £1320 „ „ $2\frac{1}{2}$?
7. 3 years hence if the true discount is £36 „ „ 4 ?
8. 5 „ „ „ „ „ £326, 5s. „ „ $4\frac{1}{2}$?

Find the **Present Worth** and **True Discount** on a bill of :—

9. £894, 18s. due 4 years hence at $3\frac{1}{2}$ per cent.
10. £23, 8s. 4d. „ 9 months „ „ $3\frac{3}{4}$ „
11. £625, 5s. „ $2\frac{1}{2}$ years „ „ $3\frac{1}{2}$ „
12. £793 „ 6 months „ „ $5\frac{1}{2}$ „
13. £911, 13s. 3d. „ 5 years „ „ 3 „
14. £1356, 13s. 4d. „ 3 months „ „ 7 „
15. £808, 1s. 4d. „ 3 years 9 months „ „ 4 „
16. £850 „ 2 years „ „ 5 „
17. £725 „ 3 years „ „ 4 „
18. £805, 10s. „ $2\frac{1}{2}$ years „ „ 3 „

When is the bill due, if the :—

19. Discount on £1250 at 5 per cent. is £250 ?
20. „ „ £630 at 3 per cent. is £67, 10s. ?
21. „ „ £2760, 15s. at 3 per cent. is £60, 15s. ?
22. Present worth on £459 at 5 per cent. is £382, 10s. ?
23. „ „ £1041, 15s. at $4\frac{1}{2}$ per cent. is £900 ?
24. „ „ £840 at $5\frac{1}{2}$ per cent. is £400 ?

What is the **rate** of interest, when the :—

25. Present worth of £269, 10s. due $2\frac{1}{2}$ years hence is £245 ?
26. „ „ £4731, 2s. due 2 years 4 months hence is £4207, 10s. ?
27. Discount on £237, 10s. due 2 years hence is £29, 3s. 4d. ?
28. „ „ £587, 18s. 9d. due 6 months hence is £12, 18s. 9d. ?
29. Present worth of £600, 15s. due $2\frac{1}{2}$ years hence is £540 ?
30. Discount on £268, 18s. 4d. due 5 years hence is £38, 8s. 4d. ?

Find the **Present Worth** and **True Discount** (reckoning *compound interest*) on a bill of :—

31. £91, 17s. 6d. due 2 years hence at 5 per cent.
32. £264, 12s. „ 2 „ „ 5 „
33. £1620, 13s. 6d. „ 4 „ „ 5 „
34. £2894, 1s. 3d. „ 3 „ „ 5 „
35. £1081, 12s. „ 2 „ „ 4 „
36. £198, 2s. 11d. „ 6 „ „ $3\frac{7}{8}$ „
37. £1000 „ 3 „ „ 3 „
38. £2500 „ 2 „ „ 4 „

Miscellaneous Examples on True Discount

Exercise LXXIII.

[*The answers are to be given correct to the nearest penny.*]

1. Find the discount on a bill of £397, 2s. due 6 months hence at 5 per cent.
2. What is the present worth of £1051, 5s. due a year hence at £5, 2s. 6d. per cent. ?
3. Find the discount on £76, 15s. due 8 months hence at $3\frac{1}{2}$ per cent.
4. When is the bill due if the discount on £4600 at 8 per cent. is £766, 13s. 4d. ?
5. What is the rate of interest when the present worth of a bill for £151, 10s. due in 80 days is £150 ?
6. Find the discount on £1456, 10s. due 3 years hence at $4\frac{1}{2}$ per cent.

7. Find the present worth of a bill of £1374, 10s. due 6 years hence at $4\frac{1}{2}$ per cent.
8. What is the discount on £3540, 10s. due 3 years hence at $4\frac{1}{2}$ per cent.?
9. What is the rate of interest when the discount on a bill of £3570, 18s. 4d. due 50 days hence is £18, 5s.?
10. When is a bill of £472, 10s. due if the discount on it at the rate of $2\frac{1}{2}$ per cent. is £52, 10s.?
11. Allowing interest at $3\frac{1}{8}$ per cent., what sum of money will now discharge a debt of £903, 11s. 3d. due 11 months hence?
12. Find the difference between the interest on £266, 13s. 4d. for 3 months at $4\frac{1}{2}$ per cent. and the discount on £83 due 15 months hence at 3 per cent.
13. What is the discount on £1025 due 2 years 8 months hence at $3\frac{1}{2}$ per cent.?
14. Find the difference between the interest and discount on £2229, 3s. 4d. due $1\frac{3}{4}$ years hence at 4 per cent.
15. Find the difference between the interest and discount on £100 due 5 years hence at 5 per cent.
16. How many years hence is a bill of £3044, 3s. due, if its present worth is £2675, the rate of interest being $4\frac{3}{5}$ per cent.?
17. What is the rate of interest if the discount on a bill of £122, 5s. due 5 months hence is £2, 5s.?
18. Find the discount on £2000 for $1\frac{3}{4}$ years at 3 per cent.
19. Find the present worth of £926, 2s. due 3 years hence at 5 per cent., compound interest.
20. Find the present worth of £2505 due 3 years hence at 5 per cent.

Ex. 4. *Find the* **banker's discount** *on a bill of* £532, 15s. 0d. *due 5 months hence at 4 per cent.*

Banker's discount = simple interest on bill.

$$= \frac{£532\frac{3}{4} \times 5 \times 4}{12 \times 100} = £\frac{2131 \times 5 \times 4}{4 \times 12 \times 100}$$

$$= £\frac{2131}{240} = £8,\ 17s.\ 7d.$$

Exercise LXXIV.

[*The answers are to be correct to the nearest penny.*]

Find the **banker's** (commercial) **discount** on :—

1. £150 due 3 years hence at 5 per cent.
2. £200, 17s. 6d. „ 6 months „ 4 „
3. £300, 6s. 8d. „ 2 years „ 3 per cent.
4. £420 „ 4 months „ $2\frac{1}{2}$ „
5. £313, 13s. 4d. „ 3 months „ $3\frac{1}{3}$ „
6. £250, 12s. 6d. „ $2\frac{1}{2}$ years „ $4\frac{1}{2}$ „

Find the **difference** between the banker's (commercial) discount and ue discount on :—

7. £750 due 2 years hence at 5 per cent.
8. £825, 10s. „ 3 years „ 4 „
9. £350, 17s. 6d. „ 3 months „ $3\frac{1}{3}$ „
0. £425, 15s. „ $2\frac{1}{2}$ years „ $3\frac{1}{2}$ „
1. £1120, 10s. due 6 months hence at $2\frac{1}{2}$ per cent.
2. £8725, 2s. 6d. due 4 months hence at $2\frac{1}{2}$ per cent.
3. How much will a banker give for a bill of £2500 which he discounts at 3 per cent. 6 months before it is due ?
4. What will the holder of a bill of £1200 receive if he discounts it at $2\frac{1}{2}$ per cent. 3 months before it is due ?
5. Find the amount of a bill due in 10 months at $3\frac{1}{2}$ per cent., if the banker's discount on it is £17, 10s.
6. What is the commercial discount on a bill whose face value is £550, and which is discounted 4 months before it is due at 5 per cent. ?
7. A merchant sells goods for £760, allowing 6 months' credit. What would be the cash value of the goods reckoning commercial discount at 4 per cent. ?

EXCHANGE

Ex. 1. *Exchange* £750, 5s. 6d. *for French money at* 25·12 *francs per* £.

25·12 francs = £1.
£750·275 = ? francs.

∴ No. of francs = 25·12 × 750·275.
= 18846·91 (correct to 2 places).

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7502·7500
   2·512
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15005·5000
 3751·3750
   75 0275
   15·0055
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18846·91
```

Ex. 2. *A merchant remits* 59720 *florins to Amsterdam by way of Paris when the exchange of London on Paris is* 25·22 *francs for* £1, *of Paris on Amsterdam* 212 *francs for* 100 *florins. How much money must be sent?*

£1 = 25·22 francs.
212 francs = 100 florins.
59720 florins = ? £.

$$\therefore \text{English money} = £\frac{59720 \times \overset{106}{\cancel{212}}}{\underset{12{\cdot}61}{\cancel{25{\cdot}22}} \times \cancel{100}}$$

$$= £\frac{6330{\cdot}32}{1{\cdot}261} = £5020{\cdot}079$$

= £5020, 1s. 7d.

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    5972
     1 6
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    5972
   35832
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  633032

                5020·0793
1·261)6330·3200
      6305
      -----
        25 32
        25 22
        -----
          1000
           883
          ----
           117
           113
          ----
             4
             4
          ----
```

Ex. 3. *Exchange* 500 *francs for English money, when English money is at a premium of* 15 *per cent., and* £1 = 25·22 *francs at par.*

$$£1 = 25{\cdot}22 + \frac{25{\cdot}22 \times 15}{100} \text{ francs.}$$

= 29·003 francs

$$\therefore 500 \text{ francs} = \frac{500}{£29{\cdot}003}$$

= £17·240

= £17, 4s. 10d.

```
                17·2396
2·9003)50·0000
       29 003
       -------
       20 9970
       20 3021
       -------
         6949
         5801
         ----
         1148
         1044
         ----
          104
           87
          ---
           17
           17
          ---
```

Exercise LXXV.

Find the **value in English money** (to the nearest penny) of :—

1. 820 francs at 25·17 francs for £1.
2. 897·9 francs at 25·21 „ „
3. 972·5 francs at 25·35 „ „
4. 712·14 Dutch florins at 12·14 florins for £1.
5. 850 reichsmarks at 20·47 reichsmarks for £1.

Find the **value in foreign money** (correct to two places of decimals) of :—

6. £50, 10s. in dollars at 4·86 dollars for £1.
7. £125, 15s. 6d. in francs at 25·22 francs for £1.
8. £340, 15s. 3d. in Dutch florins at 12·15 florins for £1.
9. £535, 2s. 6d. in milreis at 4·53 milreis for £1.
10. £976, 5s. in reichsmarks at 20·45 reichsmarks for £1.
11. If £1 = 20·45 marks, and 10 marks = 12·31 francs, express £30, 5s. in francs (correct to 2 places).
12. Exchange £45, 12s. in dollars, when £1 = 20·43 marks and 4·2 marks = 1 dollar (correct to 2 places).
13. If £1 = 25·15 francs, 12·35 francs = 10 marks, and 17·03 marks = 10 florins, find the value of £130, 15s. in florins (correct to 2 places).
14. If the exchange between London and New York is 4·87 dollars per £1, and between Paris and London 125·85 francs per £5, what is the exchange between Paris and New York? (Answer in dollars per franc to 4 decimal places.)
15. If French money is at a premium of 10 per cent., exchange 1000 francs into German marks, when 100 marks = 123·2 francs.
16. United States money being at a premium of 15 per cent., exchange 1200 marks into dollars, when 1 dollar = 4·2 marks.

17. A London merchant remits 60,000 marks to Berlin by way of Amsterdam, when the exchange of London on Amsterdam is 12·14 florins per £1, of Amsterdam on Berlin 10 marks for 5·92 florins. How much money must be sent? (Answer to the nearest penny.)

18. A Berlin firm owe a Paris firm 2750 francs, the exchange between Berlin and Paris being 100 marks for 123·4 francs. What do the firm gain or lose by delaying payment till the rate is 30 marks for 36·5 francs? (Answer in marks correct to 2 places.)

19. If £525 are remitted from London to Amsterdam in payment of an account of 6363 florins, what is the rate of exchange in shillings per florin? (Answer to 2 decimal places.)

20. A London merchant has to remit 7200 roubles to St. Petersburg when £1 is worth 6·3 roubles. Will it be more advantageous to send direct or by way of Paris and Berlin, if the exchange between London and Paris is £1 = 25 francs, between Paris and Berlin 25·2 francs = 20·4 marks, and between Berlin and St. Petersburg 20 marks = 6·15 roubles?

21. Will it be more advantageous to send 25,000 florins direct to Amsterdam or by way of Paris? Between London and Amsterdam the exchange is £1 = 12·08 florins, between London and Paris £1 = 25·15 francs and between Paris and Amsterdam 10 francs = 4·8 florins.

WORK AND PIPES

Ex. 1. *A does a piece of work in 2 days which B could do in 3 days; how long will they take when working together?*

A does $\frac{1}{2}$ of work in 1 day.

B ,, $\frac{1}{3}$,, ,,

$\therefore$ A+B do $(\frac{1}{2}+\frac{1}{3})=\frac{5}{6}$ of work in 1 day.

$\therefore$ A+B do whole work in $\frac{6}{5}=1\frac{1}{5}$ days.

Ex. 2. *One pipe fills a cistern in 25′, a second in 30′, and a third empties it in 35′. How long will it take to fill the cistern, if all the pipes be opened?*

1st pipe *fills* $\frac{1}{25}$ of cistern in 1′.

2nd ,, ,, $\frac{1}{30}$,, ,,

3rd ,, *empties* $\frac{1}{35}$,, ,,

$\therefore$ All three pipes fill $(\frac{1}{25}+\frac{1}{30}-\frac{1}{35})=\frac{42+35-30}{1050}$

$=\frac{47}{1050}$ of cistern in 1′.

$\therefore$ They fill whole cistern in $\frac{1050}{47}'=22\frac{16}{47}'$.

Exercise LXXVI.

1. A does a piece of work in 7 days which B could do in 21 days; how long will they take working together?

2. A takes 3 days over a piece of work which occupies B 5 days; how long will they take when working together?

3. If A can do in 4 hours the same piece of work which B could do in 5 hours and C in 12 hours, how long will they take if all of them work?

4. If one pipe fills a cistern in 10 minutes and another in 15 minutes, how long will it take if both of them are acting?

5. One pipe fills a cistern in 12 minutes and another in 14 minutes; how long will they take when working together?

6. If one pipe fills a cistern in 8 minutes and another empties it in minutes, how long will it take to fill the cistern if they are both opened?

7. Of two pipes, one can fill a tank in 10 minutes and the other empty i in 12 minutes. If both are working, how long will it be before th tank is full?

8. Of three pipes, one can fill a certain tank in 8 minutes, another i 9 minutes, and the third can empty it in 7 minutes. How long wil it take to fill the tank if all the pipes are opened?

9. Two pipes can fill a cistern in 5 and 6 minutes respectively. They ru together till one-half is filled, and then the first pipe is shut off what is the total time taken to fill the cistern?

10. Two pipes could fill a cistern in 7 and 8 minutes respectively, and third could empty it in 12 minutes. They run together till one-thir of the cistern is full, and then the third pipe is shut off. What i the total time taken to fill the cistern?

11. A, B, and C can do a piece of work in 4, 5, and 8 days respectively After 1 day C stops; how long will it take A and B, workin together, to finish the work?

12. A, B, and C can do a piece of work in 6, 12, and 15 days respectively After 1 day C stops, and then A and B work together for 2 mo days, when B stops; how long will it take A to finish the work?

13. If two pipes can fill a tank in 10 and 12 minutes respectively, and third pipe empty 30 cu. feet in 6 minutes, what is the capacity the tank if, when all of them work together for 2 minutes, 34 c feet are filled?

AVERAGES AND MIXTURES

Ex. 1. *Find the average of the numbers* 116, 132, 147, 185.

$$\text{Average} = \tfrac{1}{4}(116+132+147+185) = \tfrac{1}{4} \times 580 = 145.$$

Ex. 2. *The average weight of* 8 *parcels is* 25·5 *lbs. If the average of the first four is* 20·2 *lbs., and that of the last three* 34 *lbs., what is the weight of the fifth parcel?*

All the parcels weigh $25{\cdot}5 \times 8 = 204$ lbs.

First four ,, $20{\cdot}2 \times 4 = 80{\cdot}8$ lbs.

Last three ,, $34 \times 3 = 102$ lbs.

$\therefore$ *Fifth* weighs $204 - 80{\cdot}8 - 102 = 21{\cdot}2$ lbs.

Ex. 3. *If* 4 *lbs. of tea at* 4s. 2d. *per lb. are mixed with* 8 *lbs. of tea at* 5s. 11d. *per lb., at what price per lb. must the mixture be sold so as to gain* 5%?

4 lbs. at 4s. 2d. per lb. are worth 16s. 8d.

8 lbs. at 2s. 11d. ,, ,, 23s. 4d.

$\therefore$ Total value of 12 lbs. of mixture is 40s.

$\therefore$ Selling price of 12 lbs. $= 40\text{s.} \times \frac{105}{100} = 42\text{s.}$

$\therefore$ Selling price of 1 lb. $= 3\text{s. } 6\text{d.}$

Ex. 4. *In what proportion must a grocer mix teas worth* 2s., 2s. 6d., *and* 3s. *per lb., so that the mixture may be worth* 2s. 4d.?

On 1 lb. of the 1st tea put into mixture, the *gain* is 4d.

,, 2nd ,, the *loss* is 2d.

,, 3rd ,, the *loss* is 8d.

Thus if we take the teas in any of the following ways, the total gain will be equal to the total loss:—

2s. tea	3 lbs.	4	5	5	
2s. 6d. ,,	2 ,,	4	6	2	
3s. ,,	1 ,,	1	1	2	etc.

Exercise LXXVII.

Find the average of :—

1. 72, 39, 47, 85, 72.
2. 527, 823, 41, 53.
3. 825, 597, 421, 327, 51.
4. 8 cwts. 25 lbs., 2 qrs. 1 lb., 3 cwts. 3 qrs. 2 lbs., 2 qrs. 25 lbs., 1 cw 2 lbs.
5. $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{1}{4}$, $4\frac{1}{5}$.
6. $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$, $5\frac{5}{6}$.
7. £5, 4s. 6d., £3, 2s. 9d., £1, 11s. 7d., £8, 2s. 5d.
8. £4, 13s. $8\frac{1}{2}$d., £9, 10s. 7d., £8, 3s. $4\frac{1}{2}$d., £5, 0s. $10\frac{1}{2}$d., £8, 7s. $3\frac{1}{2}$d.
9. On five successive days a man cycles 52 mls. 7 fur., 63 mls. 5 fu 45 mls. 4 fur. 100 yds., 10 mls. 200 yds., and 70 mls. 5 fur. 150 yd What is his average distance per day ?
10. The weights of the Cambridge VIII. for 1903 were 11 st. $1\frac{3}{4}$ lbs., 12 s 6 lbs., 11 st. $1\frac{1}{2}$ lbs., 12 st. 11 lbs., 13 st. $4\frac{1}{4}$ lbs., 12 st. 10 lb 12 st. $6\frac{1}{2}$ lbs., and 11 st. $4\frac{1}{2}$ lbs. What was the average weight ?
11. A person buys 35 articles at 2s. 6d. each and 24 at 1s. 8d. eac What is the average price ?
12. A man buys 5 dozen of wine at 36s. per dozen, 4 dozen at 34s. p dozen, and 6 dozen at 42s. per dozen. What is the average pri per dozen ? (Answer to the nearest penny.)
13. If the height of a number of men is measured, and the average of t first six is 5 ft. 5 ins., of the next seven 5 ft. $5\frac{1}{2}$ ins., and of t next eleven 5 ft. $4\frac{3}{4}$ ins., what is the average height of the who number of men to the nearest hundredth of an inch ?
14. If 5 lbs. of tea at 1s. 9d. per lb. are mixed with 4 lbs. at 2s. 6d. p lb., what is the price of the mixture per lb. ?

15. If 17 lbs. of sugar at $3\frac{1}{2}$d. per lb. are mixed with 21 lbs. at 3d. per lb., what is the price per lb. of the mixture? (Answer to the nearest farthing.)

16. 10 lbs. of coffee at 1s. 1d. per lb. are mixed with 15 lbs. at 8d. per lb., what is the price per lb. of the mixture?

17. A merchant buys 5 dozen wine at 35s. a dozen, 4 dozen at 32s., and 3 dozen at 41s. At what price per dozen must he sell the mixture so as to gain 54s. on the transaction?

18. A grocer mixes 25 lbs. of coffee at 1s. 3d. per lb. with 20 lbs. at 1s. 7d. per lb. and sells the mixture at 1s. 6d. per lb. What does he gain on the transaction?

19. 15 lbs. of tea at 2s. 3d. per lb. are mixed with 40 lbs. of tea at 2s. 6d. per lb. Find the selling price per lb. of the mixture, if there is to be a gain of 20 per cent.

20. A grocer has equal quantities of two kinds of tea worth 5s. and 4s. per pound respectively. He takes one-third of each kind and mixes it with the other. At what price per pound ought the two mixtures to be respectively sold?

21. A coal-merchant mixes 20 tons of coal at 20s. per ton with 25 tons at 22s., and sells the mixed coal so as to gain $33\frac{1}{3}$ per cent. For what will he sell 27 tons?

22. A grocer buys teas at 1s. 2d., 1s. 4d., and 1s. 9d. per lb., and mixes them in equal proportions; what will be the price of 56 lbs. of the mixture?

23. In what proportion must a grocer mix teas worth 3s. and 2s. 6d. per lb. respectively, in order that the mixture may be worth 2s. 10d. per lb.?

24. Sugar worth $2\frac{3}{4}$d. per pound is mixed with sugar worth 2d. per lb. so that the mixture is worth $2\frac{1}{2}$d. per lb. In what proportion are they mixed?

25. A publican mixes gin worth 15s. a gallon, water and base spirit worth 10s. a gallon. He sells the mixture at 2s. 10d. per bottle of six to the gallon at a gain of $118\frac{4}{7}$ per cent.; in what proportion does he mix them?

26. A grocer has two sorts of tea, which cost him 2s. 8d. and 2s. 1d. per lb. respectively; in what ratio must he mix them so that he may gain 25 per cent. by selling the mixture at 2s. 11d. a pound?

27. In what proportions should wines worth 15s., 20s., 26s., and 30s. per gallon be mixed, so as to give a profit of $14\frac{2}{7}$ per cent. when the mixture was sold at 24s. per gallon?

28. A grocer buys some tea at 4s. per lb. and some at 5s. 6d. In what proportion must he mix them, that when he sells the tea at 6s. per lb. he may gain 20 per cent.?

29. A milk-dealer buys pure milk at $11\frac{1}{2}$d. per gallon. How much water must he add that he may sell at 5d. a quart, and obtain a gross profit of 100 per cent.?

30. A tradesman buys milk at 11d. a gallon, and after diluting it with water, sells it at $3\frac{1}{2}$d. per quart, making a profit of 50 per cent. What proportion of water did he add to the milk?

31. Taking the values of zinc and copper to be £17, 14s. and £73, 15s. per ton respectively, find how much of each metal there will be in a mass compounded of the two which weighs 14 cwts. and is worth £23, 12s.

32. If coffee and chicory cost £8, 10s. and £2, 10s. per cwt. respectively, what is the proportion of coffee and chicory in a mixture of which 7 lbs. are worth 7s. 6d.?

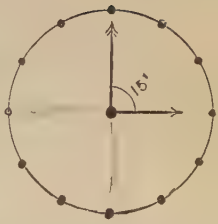
CLOCKS

The *large hand* moves 60′ while the *small hand* 5′

∴ ,, ,, 12′ ,, ,, 1′

∴ The *large hand gains* 11′ on the small hand while *it* moves . . . . . 12′

∴ **large hand gains 1′ on small hand while it moves** . . . . . $\frac{12'}{11}$



Ex. 1. *At what time between* 3 *and* 4 *o'clock are the hands of a clock* (i) *at right angles,* (ii) *coincident?*

(i) The large hand starts 15′ behind the small hand, it finishes 15′ ahead,

∴ it gains 30′ on small hand.

It does this in $\frac{12}{11} \times 30' = 32\frac{8}{11}'$.

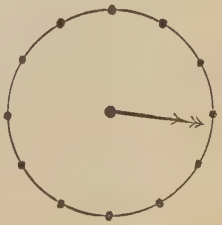
∴ The two hands are at right angles at $32\frac{8}{11}'$ past 3.

(ii) The large hand starts 15′ behind the small hand, it finishes level.

∴ it gains 15′ on the small hand.

It does this in $\frac{12}{11} \times 15' = 16\frac{4}{11}'$.

∴ The two hands are coincident at $16\frac{4}{11}'$ past 3.



Ex. 2. *A clock gains 5″ per hour. It is put right at 12 noon; what will be the correct time when it indicates 12 noon on the succeeding day?*

The clock moves 1 hour 5 secs., *i.e.* $1\frac{1}{720}$ hours, in 1 *hour of correct time.*

$\therefore$ The clock moves 24 hours in $\dfrac{24}{1\frac{1}{720}}$ *correct hours.*

$= \dfrac{24 \times 720}{721}$ hrs. $= 23$ hrs. $58\frac{2}{721}$ mins.

$\therefore$ *Correct time* is $1\frac{719}{721}$ mins. to 12.

Exercise LXXVIII.

At what times will the hands of a clock be

(i) Coincident, (ii) at right angles, (iii) opposite, (iv) 20 minutes apart, between—

1. 1 o'clock and 2 o'clock?
2. 2 o'clock and 3 o'clock?
3. 4 o'clock and 5 o'clock?
4. 7 o'clock and 8 o'clock?
5. 8 o'clock and 9 o'clock?
6. 9 o'clock and 10 o'clock?
7. A clock gains 10 seconds per hour, and is put right at 9 o'clock in the morning; when on the next day it indicates 7 A.M. what will be the correct time?
8. A clock loses 10 seconds per hour, and is put right at 8 A.M.; what will be the correct time when it indicates 8 P.M. on the same day?
9. What will be the correct time when a clock indicates 11 P.M., the clock losing 5 seconds per hour, and having been put right at 12 noon on the same day?
10. Two watches are set together. The first gains 4 minutes a week and the second loses 4 minutes a week; how long will it be before they indicate the same time?
11. Two watches are set together. The first gains 5 minutes a week and the second 3 minutes a week; how long will it be before they indicate the same time?
12. A clock which is right at 9 A.M. indicates 4 minutes past 10 when it should indicate 10 A.M.; assuming that it gains uniformly, find the true time when the clock indicates 5 P.M. on the same day.

13. A watch loses 3 minutes in 24 hours; it is set right at noon on Monday: what is the correct time when it indicates $1\frac{1}{2}$ minutes past one o'clock on the following Wednesday afternoon?

14. A clock set right at noon on Tuesday loses at the rate of 192 seconds in 10 hours. What is the true time on the following Friday afternoon when the reading of the clock is 2 hours 36 minutes?

15. Of two watches, one loses 5 seconds and the other gains 4 seconds in 12 hours; they are both right at noon on a certain day; when will one be 5 minutes before the other, and what time will each then show?

To reduce a recurring decimal to a vulgar fraction.

(i) Let $F = \cdot\dot{1}5\dot{3}$.

$\therefore\ 1000\,F = 153\cdot\dot{1}5\dot{3}$.

$\therefore\ 999\,F = 153$.

$$F = \frac{153}{999} = \frac{17}{111}.$$

(ii) Let $F = \cdot 01\dot{3}4\dot{7}$.

$\therefore\ 100000\,F = 1347\cdot\dot{3}4\dot{7}$.

$100\,F = 1\cdot\dot{3}4\dot{7}$.

$\therefore\ 99900\,F = 1346$.

$$F = \frac{1346}{99900} = \frac{673}{49950}.$$

Rule 1. **To reduce a pure recurring decimal to a vulgar fraction, place all the figures to the right of the decimal point in the numerator, and in the denominator as many nines as there are figures to the right of the decimal point.**

Rule 2. **To reduce a mixed recurring decimal to a vulgar fraction, take as numerator the excess of all the figures over the non-recurring figures, and as denominator as many nines as there are recurring figures and as many ciphers as there are non-recurring figures to the right of the decimal point.**

Ex. 1. $\cdot\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$.

Ex. 2. $3\cdot 1\dot{7} = 3\frac{16}{90} = 3\frac{8}{45}$.

Ex. 3. Reduce $\cdot 02\dot{7}14285$ to a fraction.

Let $F = \cdot 02\dot{7}14285$.

$\therefore\ 100\,F = 2\cdot\dot{7}14285 = 2\frac{5}{7} = \frac{19}{7}$.

$\therefore\ F = \frac{19}{700}$.

Exercise LXXIX.

Express as vulgar fractions in their lowest terms :—

1. $\cdot\dot{0}3\dot{7}$, $\cdot\dot{5}\dot{5}$, $\cdot\dot{4}\dot{4}$, $\cdot\dot{7}\dot{3}$.
2. $\cdot 5\dot{2}\dot{5}$, $\cdot 0\dot{7}\dot{6}$, $\cdot 1\dot{3}\dot{5}$, $\cdot 1\dot{5}\dot{6}$.
3. $\cdot 012\dot{3}\dot{6}$, $\cdot\dot{3}12\dot{3}$, $3\cdot 017\dot{4}\dot{8}$, $\cdot\dot{5}\dot{4}$, $5\cdot\dot{9}$.
4. $\cdot\dot{2}5\dot{9}$, $5\cdot\dot{6}\dot{8}\dot{1}$, $2\cdot\dot{6}\dot{0}\dot{8}$, $5\cdot\dot{6}$.
5. $2\cdot 50\dot{8}$, $\cdot 00\dot{4}9\dot{5}$, $3\cdot 40\dot{1}8\dot{5}$.
6. $\cdot 09\dot{6}\dot{3}$, $5\cdot 90\dot{4}0\dot{5}$, $9\cdot 03\dot{2}\dot{8}$.
7. $\cdot 003\dot{1}42857\dot{1}$, $5\cdot 0\dot{2}85714$, $10\cdot\dot{8}57142$.
8. $\cdot 02\dot{1}53846$, $5\cdot 20\dot{6}92307$, $\cdot 01\dot{8}46153$.

MISCELLANEOUS PROBLEMS

Exercise LXXX.

1. By selling a horse for £89, 12s. 0d., I lose 20% on what he cost me ; for how much must I sell him to gain 10% on what he cost me ?

2. A piece of work can be done by 3 men and 4 boys in 6 days, by 3 men and 1 boy in 8 days, by 2 women and 4 boys in 10 days. How long would a woman take to do the work single handed ?

3. A person invests half his capital in the 3 per cents. at 91, and half in the 4 per cents. at par. Which investment produces the larger income ? Supposing his total income to be £500, what is his capital ?

4. A works 7 hours a day for 6 days, B 4 hours a day for 9 days, and C 5 hours a day for 7 days ; if the wages paid to A and B together amount to £7, 16s. 0d., what do A, B, and C receive ?

5. The front seats at a concert cost half-a-crown each, and the back seats a shilling. If 400 tickets are sold and the receipts are £30, 10s. 0d., how many front seats are taken?

6. Taking the value of gold as 3 francs 10 centimes per gram, find its value in English money per ounce Troy, assuming that 1 gram is 15·5 grains, and that £1 is worth 25·25 francs.

7. A contractor undertakes to build a wall 1000 yards long in 50 days. He employs 56 men, but at the end of 27 days finds that only 448 yards of wall are built. How many extra men must he employ in order that the wall may be finished in time?

8. A tradesman marks his goods at 20% above cost price; if he then allows a discount of 1s. 3d. in the £ off the marked price, what percentage of profit does he make?

9. Find by contracted methods the value of
$35{\cdot}705682 \times 581{\cdot}35823 \div 82{\cdot}05972$ correct to three places of decimals.

10. A landlord spends $\frac{1}{6}$ of the gross rent of a house in repairs, and pays income tax at 11d. in the £ on the balance. If he then have £85, 17s. 6d. left, what was the gross rent?

11. An open rectangular cistern, of which the internal length, breadth, and depth are as 4 : 3 : 2, is made of metal, 1·25 centimetres thick and 8 times as heavy as water. If the capacity of the cistern is 1265·625 litres, what is its weight in kilograms when empty? (1 cu. cm. of water weighs 1 gm.)

12. Find the cost of excavating a ditch half a mile long, $7\frac{1}{2}$ ft. wide, and 4 feet deep, at the rate of 6d. per cubic yard for the first foot in depth, the rate increasing 1d. per cubic yard for each foot in depth.

13. A person leaves his estate valued at £14,500 to his widow and three children, so that, after estate duty has been paid, the widow shall receive double as much as a child. If estate duty is at the rate of 5 per cent., find what each receives.

14. Three quarters of an estate is bought for £15 an acre and the rest at £20 an acre. By selling the whole at £19 an acre a profit of £1000 is made. Find to the nearest acre the size of the estate.

15. If 7 per cent. of the sale price of an article is equal to 8 per cent. of its cost price, and 9 per cent. of the sale price exceeds 10 per cent. of the cost price by half-a-crown, at what prices is the article bought and sold?

16. German silver consists of 3 parts by weight of zinc, 2 of nickel, and 5 of copper. The only nickel obtainable is that in an alloy consisting of 7 parts of zinc, 5 of nickel, and 3 of copper. What quantity of this should be taken, and how much zinc and copper should be added, to make 25 lbs. of German silver?

17. A capitalist lays out £12,696 in the purchase of a 3 per cent. stock, and derives therefrom an income of £360. If the expenses of purchase are at the rate of £4 for every £500 stock, what was the price at which he purchased?

18. A metre being 39·37 inches, and a hectare the area of a square whose side is 100 metres, express a hectare in acres (correct to 2 decimal places).

19. A motor car was timed over a measured quarter mile, and the time given as 34 seconds. The error made in taking the time may have been 1 second. Within what limits does the speed (in miles an hour) lie?

20. A sovereign weighs $123\frac{1}{3}$ grains and $\frac{11}{12}$ of this weight is gold; whilst a shilling weighs $87\frac{1}{4}$ grains and $\frac{37}{40}$ of this weight is silver. If gold is worth £4, 5s. 0d. per oz. of 480 grains, and silver is worth 2s. 1d. per oz., find the value of the gold in a sovereign and the value of the silver in a shilling.

21. The Scotch Education Department requires for each pupil in a school-room 8 sq. ft. of floor space and 80 cu. ft. of air space. How many pupils may use a room 40 ft. long, 30 ft. wide, and 12 ft. high?

22. The whole revenue of the United Kingdom in the year 1901-2 was £143,581,542, and the revenue from income tax was £35,378,700. If the revenue from income tax is 25% less for the year 1903-4, and from other sources 10% more, what will be the revenue for the year to the nearest pound? And what percentage (correct to 2 places of decimals) of the whole revenue is then furnished by the income tax?

23. Find (correct to 3 places of decimals) the percentage error made in reckoning a pound $\frac{5}{11}$ of a kilogram; having given that an ounce is 28·35 grams.

24. How many cubes each $3\frac{1}{2}$ inches to the edge can be cut from a block of wood 16 inches long, 12 inches wide, and 9 inches thick? How many such cubes could be made from a similar block of metal, which could be melted down and recast?

25. Find the weight in ounces of the quantity of rain which could cover an acre to the uniform depth of one inch, supposing a cubic foot of rain-water to weigh 1000 ounces.

Show that this weight is more than 101 tons.

26. A mounted troop went into battle, each man riding one horse and leading another. They were defeated and fled on horseback, leaving 17 men and 30 horses dead, while 80 horses were captured by the enemy. Assuming that each man in his flight had only the horse he rode, determine how many men were originally in the troop.

27. Goods are sold so that when 10 per cent. is allowed off the sale price a profit of $12\frac{1}{2}$ per cent. is made. How much per cent. is the sale price higher than the cost price?

28. A person invests £6030 in India 3 per cents. at $100\frac{1}{2}$, and pays income tax at 1s. 2d. in the pound; on the stock rising to 102 he sells out and invests the proceeds in railway stock at 180, paying 5 per cent. free of income tax. Find the change in his net income.

29. December 1, 1902, was on Monday. On what day of the week will January 1, 1920 fall?

There are 97 Leap years in 400 Civil years. Show that the average duration of a Civil year exceeds the Solar year of 365 days 5 hrs. 48 min. 46 sec. by 26 sec.

30. A lidless cistern, whose internal length, breadth, and height are 3 ft. 6 in., 2 ft. 10 in., and 2 ft. 4 in. respectively, is made by riveting together rectangular slabs of slate of one inch thickness. Find the weight (in lbs.) of the slate used, having given that 1 cu. ft. of slate weighs 130 lbs.

31. In a certain country the birth-rate per annum is 41, and the death-ra 25, for every 1000 inhabitants at the beginning of the year. If t population at the beginning of a certain year is four millions, find the nearest thousand what it was three years earlier, assuming immigration or emigration.

32. A profit of 20 per cent. is made on goods when a discount of 10 p cent. is given on the list price. What profit per cent. will be ma when a discount of 20 per cent. is given on the list price?

33. A man invests a sum of money in 3 per cent. stock at 99. If he h invested in the same stock when the price was 98, he would ha received an income £2, 10s. greater. How much did he invest?

34. A besieged garrison have food which will last for 60 days at the ra of 20 oz. a day for each man. They are reinforced by an additi of 864 men, and the food will now last for only 55 days at 16 oz. day for each man. How large was the original garrison?

35. Find the weight in kilograms of an iron girder which is 5·4 metr long, ·54 metres wide, and ·45 metres thick, having given that cubic centimetre of iron weighs 7·76 grams.

36. A wine merchant mixed 45 gallons of one kind of wine with $22\frac{1}{2}$ gallo of another kind and sold the mixture at 14s. 8d. a gallon, th making a profit of $33\frac{1}{3}$ per cent. If a gallon of the first kind wine cost 12s., how much did a gallon of the second kind cost?

37. The pressure exerted by the atmosphere is 15 lbs. per sq. inch. E press this pressure in grams per sq. cm. correct to the nearest gra [1 centimetre = ·39 inches ; 1 gm. = ·0022 lbs.]

38. Two passengers have between them 345 lbs. of luggage, and pay their excess luggage 4s. 2d. and 6s. 8d. respectively. If the lugga had belonged to one of them the excess charge would have been 1 How much free luggage is allowed to each passenger?

39. Of two clocks, one is gaining at the rate of one minute in 15 hou and the other losing at the rate of five minutes in a week. Tuesday afternoon at half-past six, the former is 5 minutes slow, a the latter 4 minutes fast. On what day and at what time will th

be in exact agreement, and for how long will the difference between them be less than $1\frac{1}{2}$ minutes?

40. On what principal will the interest amount to 835·38 francs in 4 years at 10% per annum compound interest? Give the answer in francs.

41. A barometer tube, whose internal diameter is 2 centimetres, is 120 cms. long and is filled with mercury. Find the weight of the mercury, being given that 1 cu. cm. of mercury weighs 13·6 gms., and that the area of a circle is $0{\cdot}785 \times (\text{diameter})^2$. Also find (to the nearest penny) the cost of the mercury at 6s. 3d. a Kg.

42. A carriage starts at 2.15 P.M. Half an hour afterwards a cyclist starts to overtake it, and at 3.45, when he has just ridden 10 miles, he sees the carriage half a mile ahead. At what pace was the carriage travelling?

43. The capital of a company is five millions. The average weekly income is £18,574 and the average weekly expenditure £12,818. £150,000 a year is set aside for depreciation and the rest is divided among the shareholders. What interest per cent. yearly will the shareholders get for their money? [Answer correct to 2 decimal places.]

44. The volume of a wedge which has a rectangular base is $(2l + e)\frac{bh}{6}$ cubic inches, where e is the length in inches of the edge, h the height in inches of the wedge, l and b the length and breadth in inches of the base. Find, to the nearest cubic inch, the volume of a wedge whose height is 13 inches and edge 35 inches, and the base of which is 15 inches by 8 inches.

45. A fleet that can steam $10\frac{1}{2}$ mls. / hr. on the average has to relieve a blockaded port 6000 miles away. If it is estimated that the port can hold out 22 days, will the fleet be in time? What time has it to spare, or how much late will it be?

46. The weight of a certain volume of a metal is 19·36 of the weight of an equal volume of water. Find in kilograms the weight of a bar of this metal 10 cm. by 30 mm. by 25 mm.

N.B.—1 cu. cm. of water weighs 1 gram.

47. A room is 29 ft. long, 19 ft. broad, and 14 ft. high. What will it co (to the nearest penny) to distemper the walls at 2d. per square yar allowing for one door 7 ft. by 3 ft. 6 in., a fireplace 5 ft. by 4 f and four circular window openings, each 5 ft. in diameter. [T area of a circle $= 0{\cdot}785 \times (\text{diameter})^2$.]

48. A company pays dividend on a capital of £5,000,000, the rate per cer being always a multiple of $\frac{1}{4}$. One year they have £249,419 ava able for the payment of dividend. What is the highest rate p cent. they can pay, and what sum is left over?

49. The late Cecil Rhodes founded 60 Colonial Scholarships of £300 p annum, 104 American Scholarships of £300 per annum, and German Scholarships of £250 per annum. What must be t capital sum which, at 3 per cent. per annum, will produce annual the money for these Scholarships, allowing a further £1500 a ye for management?

50. At what o'clock will a train, which leaves London for Crewe 11.0 P.M., and goes at the rate of 45 mls. / hr., meet a train whi leaves Crewe for London at 11.40 P.M., and goes at the rate 35 mls. / hr., the distance between London and Crewe being 1 miles?

51. A man holds £11,356, 14s. 8d. in G.N.R. 4 per cent. Debentur Find his half-yearly income; find also the amount which will deducted for income tax at 1s. 3d. in the pound. [Answer to t nearest penny.]

52. Two men were walking, each at 3 mls. / hr., in opposite directions the side of a railway, and a train passed one of the men in 5 secor and the other in 6 seconds. Find the length of the train.

53. The manufacturer of a machine sells it to a wholesale dealer, makin profit of 20% on its cost; the wholesale dealer sells it to a sh keeper, making a profit of 5%; and the shopkeeper makes a pr of 12% by selling it for £79, 0s. 3d. Find, to the nearest pen the cost to the manufacturer.

54. A man sold some 3 per cent. stock at 90, and invested the money realised, together with £300 more, in 4 per cent. stock at 104. If the man's net income, after income tax at 1s. 2d. in the pound had been deducted in each case, was thus increased by £28, 5s., how much stock was sold?

55. The receipts of a company amounted in one year to £354,000 and the expenses to £251,000. In the subsequent year receipts fell off by $3\frac{1}{2}\%$ and expenses increased $\frac{1}{2}\%$. Find by what percentage of itself the profit in the first year exceeded the profit in the succeeding year.

56. The total number of marks obtainable in a certain examination is 6200, and this number is distributed between Latin, Greek, and Mathematics, in the ratio of the numbers $2\frac{1}{2}$, $2\frac{1}{4}$, and 3. A candidate who made altogether 3996 marks, made in Mathematics alone 10 more than he did for Latin and Greek together. If his marks for Greek amounted to 31% of the total awarded for that subject, how many marks did he make in each subject?

57. Find, from the data given in the following table, the percentage of the increase of the population in urban districts of England and Wales, in (1) districts of 100,000 inhabitants or over, (ii) districts of under 100,000 inhabitants, (iii) all urban districts for the ten years 1891-1901.

Give the answers to the nearest integer.

Urban Districts with Population of	Aggregate Population.	
	1891.	1901.
250,000 and upwards	7,293,005	7,972,790
100,000 and under 250,000	2,987,836	3,517,912
50,000 and under 100,000	2,448,704	3,015,571
20,000 and under 50,000	3,683,501	4,433,823
10,000 and under 20,000	2,538,281	3,006,280
3,000 and under 10,000	2,396,204	2,693,230
Under 3000	396,446	414,664

58. The volume of a sphere is 4·189 times the cube of the radius. Fin (correct to $\frac{1}{100}$ of a lb.) the weight of a hollow spherical shell $\frac{1}{2}$ inc thick and 9 inches diameter inside, which is made of iron weighin 470 lb. per cubic foot.

59. A mill-wheel is supplied with water which runs through rectangular trough 2 ft. 6 in. wide to a depth of 10 inches a a speed of 4 mls. / hr. How many gallons of water (correct t 1 place of decimals) are delivered to the wheel each minute 1 cu. ft. = 6·23 gallons.

60. A man's gross income in 1902 was 20 per cent. greater than in 190 and in both years £150 of his income was exempt from income ta the tax on the rest of his income being 14d. in the £ for 1901 an 15d. in the £ for 1902. Also the amount of income tax paid f 1902 exceeded the amount for 1901 by £6, 4s. 2d. What was th man's gross income in 1901?

TEST PAPERS

[To the end of H.C.F., L.C.M., Square and Cube Root by Factors]

PAPER I.

1. Subtract forty-two thousand three hundred and two from one million six hundred. Express the answer in figures.
2. Multiply (i) 32791 by 1007.
   (ii) 31478 by 125.
3. Divide 358190 by 35, by short division.
4. Divide 345987 by 127.
5. Find the prime factors of 648, 900, and 19404.
6. Find by factors (i) the H.C.F. of 357 and 462.
   (ii) the L.C.M. of 105, 110, 231.
7. Find by factors (i) the square root of 225.
   (ii) the cube root of 27000.

PAPER II.

1. Express the difference between one million seven hundred thousand and eight, and five hundred and sixty thousand three hundred and two, in figures.
2. Multiply (i) 23491 by 1242.
   (ii) 300927 by 999.
3. Divide 324979 by 72, by short division.
4. Divide 302718 by 239.
5. Find the prime factors of 2450 and 136125.
6. Find by factors (i) the H.C.F. of 78, 330, 2310.
   (ii) the L.C.M. of 165, 429, 510.
7. Find by factors the square root of 23716.

PAPER III.

1. Express the excess of three million five thousand seven hundred an twenty-one over five hundred and twenty-two thousand thr hundred and four, in figures.
2. Multiply (i) 5009 by 410.
   (ii) 2468 by 3057.
3. Divide (i) 74257 by 168, by short division.
   (ii) 360555642 by 789.
4. Divide the product of 2035674 and 396 by 198.
5. The sum of two numbers is 197 and one of them is 82; what is the othe
6. A wall is built with 15 layers of bricks, each layer containing in bread 3 and in length 825 bricks; how many bricks were used altogethe
7. A man receives a salary of £600 a year, and pays £82 in rent, £31 f clothing, £305 for household expenses, and £82 for extras. Ho much does he save in 5 years?

PAPER IV.

1. If a garrison of soldiers eat seven thousand eight hundred and ninety-s pounds of bread in one day, how many pounds will they eat in thr hundred and sixty-five days?
2. Divide the product of 827 and 925 by 25.
3. Divide 897462 by 385, by short division.
4. Subtract the cube of 25 from the square of 155.
5. Find the greatest number that will divide 756 and 1449 exactly.
6. Find the L.C.M. of 8, 9, 16, 24, 27.
7. Find the value of $2^2+3^2+4^2+5^2+6^2$.

PAPER V.

1. Find the value of $555804 \div 12$.
2. Multiply (i) 83495 by 125.
   (ii) 82307 by 428.
3. Find the value of $51^3 - 20^3$.
4. Find the prime factors of 3969, 10241, and 2145.
5. Find the sum of all the primes between 50 and 70.
6. What is the least length which will contain lengths of 7, 15, 21, 25 yards exactly?
7. Find by factors the cube root of 166375.

PAPER VI.

1. Multiply one hundred and forty-seven thousand two hundred and ninety-seven by six thousand seven hundred and eighty-nine.
2. Find the number which, subtracted from eighty thousand, leaves fifty-seven thousand seven hundred and thirty-five, and divide it by 365.
3. Divide the product of 2468 and 3057 by 3702.
4. Find the prime factors of 2016, 2112, and 8320.
5. Divide 879201 by 1155, by short division.
6. A man buys 52 articles at 2s. each and 49 articles of another kind, the whole costing 25ls.; what was the price of each article of the second kind?
7. Add together the sum and difference of 5289 and 7247.

PAPER VII.

1. Multiply one hundred million one hundred thousand and ten by five thousand and five.
2. Find the difference between the squares of 89 and 87.
3. Divide 82794 by 231, by short division.
4. Calculate the value of $3^2 + 5^2 + 7^2 + 9^2 + 11^2$.
5. A certain number divided by 243 gives a quotient of 1306 and remainder 92; find the number.
6. Multiply 81375 by 997.
7. Find the prime factors of 2793, 7581, and 805.

PAPER VIII.

1. Multiply thirty millions three hundred and forty thousand and seven by seven thousand eight hundred and forty-six.
2. Divide 732540408423 by 273.
3. Find the prime factors of 819, 9639, and 678699.
4. A plantation has 85 rows of trees with 4231 trees in each row; how many trees are there?
5. Find the value of $549^2 - 540^2$.
6. Find the number which divided by 837 gives a quotient 25 and remainder 132.
7. Find the cube root of 12326391.

[To the end of Division of Decimals]

PAPER IX.

1. A boy saves 2s. 9d. every week. What will his savings amount to in 11 weeks?
2. Divide (i) 570021 by 77, by short division.
   (ii) the product of 608872 and 69 by 92.
3. A bar of metal weighing 100 ozs. 16 dwts. is made into coins weighing 1 oz. 8 dwts. each. How many coins are there?
4. Reduce £87, 12s. 9½d. to halfpence.
5. How often is £1, 17s. 6d. contained in £28, 2s. 6d.?
6. How many days (including both extreme days) are there between—
   (i) Jan. 1st, 1900, and Oct. 27th, 1900?
   (ii) Jan. 5th, 1896, and Feb. 2nd, 1897?
7. If in a certain business there are used during the year 1841 halfpenny, 5025 penny, and 425 twopenny stamps, what sum of money does this represent?

PAPER X.

1. Find the L.C.M. of 54, 24, 36, 45, 120, 135, 72, 15.
2. Twenty railway wheels together weigh 2 tons 15 cwts. 1 qr. 2 lbs.; what is the weight of each?
3. Reduce 5 tons 3 cwts. 2 qrs. to pounds.
4. A man bought 2 acres of ground. On part of it, measuring 25 poles, he built a house, and the rest was divided between a garden of 2 roods 20 poles and an orchard. Find the size of the orchard.
5. Find the difference between the squares of 576 and 444.
6. How many parcels of 4 lbs. 7 ozs. of tea can be made from a chest containing 3 cwts. 3 qrs. 10 lbs. 7 ozs.?
7. Divide 32471 by 35, by short division, and find by what number 638163 must be divided to give 731 as quotient.

PAPER XI.

1. Reduce 3 tons 7 cwts. 1 qr. to grains.
2. If £1468, 16s. 3d. is divided equally among 426 people, how much will each receive?
3. What duty will a merchant pay on a ton of tea at 4d. a lb.?
4. A man bicycles a distance of 200 miles. The first day he rides for 6 hours at $8\frac{1}{4}$ mls./hr., the second day for $6\frac{1}{2}$ hours at 9 mls./hr., and on the third for 5 hours and 20 minutes at $10\frac{1}{2}$ mls./hr. How long does it take him to finish the distance at the rate of 8 mls./hr.?
5. Find the cost of 135 articles at £3, 2s. 7d. each.
6. What do 2 hectolitres 50 litres of milk cost at $3\frac{1}{2}$d. a litre?
7. Simplify $10{\cdot}35 - {\cdot}67099 + {\cdot}002 - 3{\cdot}6091 + 3{\cdot}854$.

PAPER XII.

1. Reduce 1 ton 17 cwts. 3 qrs. 7 lbs. to pounds, and find the number of half-crowns in 105 guineas.
2. Multiply $35{\cdot}623$ by $\cdot00125$ and divide $3{\cdot}015$ by $\cdot012$.
3. When eggs cost 30 centimes apiece, how many eggs can I buy for 8 francs 40 centimes?
4. Add one million seven hundred and five thousand and eighteen to three hundred and seventy-nine thousand and fifty-seven.
   Divide 65182926 by 467.
5. Divide £118539, 1s. $10\frac{1}{2}$d. by 693.
6. Find the value of $30075^2 - 10025^2$.
7. A truck is loaded with 100 sacks; the sack itself in each case weighs 6 lbs. 11 ozs., and each sack contains 85 lbs. of grain. What is the weight of the whole load in tons, cwts., etc.?

PAPER XIII.

1. Find the H.C.F. of 546, 624, 676.
2. A piece of land is divided into 63 equal lots of 6 ac. 3 rds. 17 po. Find the size of the piece of land.
3. A consignment of flour consists of 789 sacks, each sack weighing 1 cwt. 3 qrs. 10 lbs. 8 ozs. What is the total weight of the whole consignment?
4. Multiply 12·36 by 2·007 and divide 798·1 by ·23.
5. What is the price of ribbon per metre, when 303 metres 20 centimetres cost 197 francs 8 centimes?
6. £368, 16s. 9d. was divided equally amongst a certain number of persons, and the share of each was £3, 4s. 8½d. How many persons were there?
7. Find the value of (i) $597^2 - 594^2$.
   (ii) $625^2 - 620^2$.

PAPER XIV.

1. How many halfpennies are there in £70181, 10s. 3½d.? Taking the width of a halfpenny as 1 inch, how far would this number of halfpennies stretch if placed in a straight line so that each coin touched those next to it?
2. Butter costs £135, 6s. 8d. per ton. What does a pound cost?
3. Among how many persons is £86, 10s. 3d. divided when the share of each is £3, 4s. 1d.?
4. Divide 614·50824 by ·010201.
5. Add together 4·317, ·0123, 31·241, and 3·4071; and divide the result by 5·062.
6. Find the value of $10{\cdot}24 \times {\cdot}10071$.
7. Find the cost of 97 m. 4 cm. at 3 francs 75 centimes per metre.

PAPER XV.

1. Divide (i) ·0468 by 29·25 ; (ii) ·055757592 by ·009207.
2. Given that a metre is 39·37079 inches, prove that the difference between 5 miles and 8 kilometres is nearly 51 yards.
3. What is the greatest number of persons who can each receive 13s. 6d. out of £100, and how much will be left after the distribution?
4. Multiply (i) 35·85 by 2·09 ; (ii) 3·585 by ·00209.
5. Find the L.C.M. of 10, 12, 14, 15, 27, 35.

   The English, Flemish, and French ells contain respectively 45, 27, and 54 inches. What is the least length exceeding 20 yards which will contain an exact number of each measure?
6. How many rails, each weighing 3 qrs. 9 lbs. 8 ozs. can be made out of 18 tons 14 cwts. of iron?
7. Eight bells begin tolling together, and toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively; how often would they all toll together again in the course of 7 hours?

PAPER XVI.

1. Simplify $\frac{6{\cdot}1275 \times {\cdot}032}{{\cdot}00024}$.
2. How many times is $11\frac{1}{4}$d. contained in £28, 2s. 6d.?
3. Divide £443716, 2s. 1d. into 817 equal parts.
4. Find, correct to the nearest centime, the cost of 287 metres 7 centimetres at 5 francs 70 centimes per metre.
5. Divide 54076539 by 63, using the method of factors and short division.
6. In an exercise in division, the divisor is 7564, the quotient 420, and the remainder 5199 ; find the dividend.
7. If 49 bushels of wheat cost £16, 1s. 5d., what would 84 bushels cost at the same rate?

10 8 5

887432 0 443716

[Including Simple Areas and Volumes, Vulgar Fractions, Decimalisation of Money]

PAPER XVII.

1. Add together $\frac{9}{77}$, $2\frac{7}{33}$, $\frac{2}{6}\frac{3}{3}$, and $7\frac{10}{11}$.
2. Find a fraction with 364 as denominator which is less than unity by $\frac{2}{13}$.
3. Add together $\frac{1}{3}$ of half-a-crown, $\frac{1}{6}$ of a guinea, $\frac{4}{15}$ of £1, and $\frac{1}{75}$ of £1, 5s. 0d.
4. After $\frac{13}{21}$ of a plank had been cut off, the length of the remainder was 5 ft. $2\frac{6}{7}$ ins. What was the length of the whole plank?
5. Reduce £3, 13s. 10d. to the fraction of £361, 15s. 8d.
6. Divide ·040548 by ·000125.
7. It requires 1344 tiles, each 9 ins. by $4\frac{1}{2}$ ins., to cover a courtyard. What is its area in sq. yds.?

PAPER XVIII.

1. Simplify (i) $\frac{7}{15}+\frac{9}{17}+\frac{13}{25}+\frac{23}{24}$.
   (ii) $8\frac{1}{2}-4\frac{2}{3}+\frac{3}{5}-1\frac{1}{6}$ of $\frac{2}{5}$.
2. Simplify $17\frac{1}{9}\div 3\frac{3}{10}-1\frac{1}{6}$ of $\frac{5}{12}$.
3. Find the value of $\frac{8}{11}$ of £87, 9s. 6d.
4. A man left £1280 to be divided among his four sons. The eldest had $\frac{5}{16}$; the second $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{1}{2}\frac{5}{8}$; the third $\frac{1}{4}$ of $\frac{7}{8}$. How much money had the youngest?
5. Multiply (i) 40·065 by 3·204; (ii) 542·1875 by 26·8.
6. Calculate the height, in metres, of a rectangular block of stone containing 245·7 cu. metres, which is 7·5 metres long and 6·3 metres broad.
7. Reduce 3s. $3\frac{3}{4}$d. to the decimal of 2s. 6d.

PAPER XIX.

1. Simplify (i) $10 + 7\frac{4}{9} + {}_{2}^{5}7 + 4\frac{2}{3} + (3\frac{1}{3} \text{ of } \frac{1}{5})$.
   (ii) $\frac{1}{7}$ of $\frac{2}{3}$ of $18 \div \frac{5}{6}$ of $\frac{7}{10}$.
2. Find the value of $\frac{2}{5}$ of £33, 5s. 10d. $- {}_{11}^{4}$ of £31, 2s. 5d.
3. Multiply £3, 12s. $1\frac{1}{4}$d. by 625.
4. Find the vulgar fraction equivalent to ·06275.
5. A man starts on a bicycle tour with a £5 note, two sovereigns, and some small change. He spends half of his money in lodgings, a third in refreshments, and $\frac{1}{12}$ in repairs to his machine, and has 12s. $5\frac{1}{2}$d. left. How much small change has he at the start?
6. Find the value of ·123725 of a ton and express the result as the decimal of 1 cwt.
7. Part of the site of Christ's Hospital, 67680 sq. ft. in area, was sold to St. Bartholomew's Hospital for £238,781. Find the price per acre to the nearest pound.

PAPER XX.

1. Find the value of $\frac{4}{15}$ of a crown, and express $5\frac{1}{4}$ sq. feet as the fraction of 7 sq. yards.
2. A father leaves £1191, 13s. 4d. to be divided equally between his seven sons and six daughters. How much will each receive?
3. Arrange the fractions $\frac{33}{63}$, $\frac{7}{12}$, $\frac{43}{84}$, $\frac{4}{7}$, in descending order of magnitude and find their sum.
4. Divide (i) ·1154255 by ·00115; (ii) 26·751 by ·000925.
5. How many persons can be paid 11s. 6d. each out of a sum of £170, 10s. 2d.? And if the balance be also distributed equally among them, how much more will each receive?
6. Reduce (i) $\frac{63}{64}$ to a decimal; (ii) ·00015625 to a vulgar fraction.
7. Reduce 2 kilograms 17 grams to the decimal of 50 kilograms 425 grams

PAPER XXI.

1. Arrange the fractions $\frac{17}{75}$, $\frac{23}{81}$, $\frac{11}{45}$, $\frac{43}{180}$ in ascending order of magnitude. Simplify $11\frac{1}{2}$ of $3\frac{2}{3} - 2\frac{3}{4}$ of $5\frac{17}{33}$.
2. Find the cost of 765 tons at £2, 17s. 8d. per ton.
3. Divide 7·04 by ·0847 to four places of decimals.
4. Express 17 cwts. 3 qrs. 21 lbs. as the decimal of a ton.
5. Find, to the nearest centime, the price of a carpet 6 metres 24 centimetres long, and 4 metres 35 centimetres broad, at 8 francs 5 centimes per sq. metre.
6. If I pay a cabdriver one penny per minute, and he drives $3\frac{1}{2}$ miles in half an hour, at what rate do I pay him per mile?
7. Find the value of £2·66875 + 16·51725s. + 1·293d.

PAPER XXII.

1. How many times would a rod 2 yds. 1 ft. $5\frac{1}{2}$ ins. be used in measuring a length of 1 fur. 108 yds. 6 ins.?
2. Reduce $3\frac{1}{7}$ of £4, 11s. 0d. to the fraction of $13\frac{3}{4}$ guineas.
3. Simplify (i) $\frac{11}{12}\frac{9}{5}\frac{6}{5}$; (ii) $3\frac{1}{7} - 2\frac{1}{15} + 4\frac{5}{21} - 1\frac{8}{35}$;
   (iii) $3\frac{1}{143} \times 4\frac{19}{17} \div 1\frac{28}{187}$.
4. Find the L.C.M. of 3875, 1160, 17500, 3784.
5. Find the value of ·052 × 1·87 × ·0021, and divide the result by the value of 3·5 × 6·63 × 1·1.
6. How many pieces of wire, each 4·13 inches long, can be cut from a rod whose length is 8·24 feet, and what will be the length of the piece left over?
7. Having given that a metre is 39·3708 inches, show that the difference between 35 yards and 32 metres is less than a centimetre.

PAPER XXIII.

1. Express £1, 7s. 9d. as the decimal of £3.
2. How many planks 18 ft. long and 8 inches wide are required t make a platform 36 yards long and 15 yards wide?
3. How many litres of water are required to fill a cistern 2 metres dm. long and 1 metre 6 dm. wide to a depth of 8 dm. 5 cm.?
4. Find the value of $2\frac{1}{7}$ of $3\frac{1}{9}$ of 4 lbs. 8 oz. 10 dwts. 12 grs.
5. Simplify $\frac{2\frac{5}{9} \text{ of } 1\frac{17}{46} - 3}{4\frac{2}{11} - 2\frac{3}{4}} \div 2\frac{2}{21}$.
6. Divide ·000324 by (i) ·018 ; (ii) ·0000018 ; (iii) 1800.
7. How many more steps does a man whose length of step is $31\frac{2}{3}$ inche take in $4\frac{3}{4}$ miles than another whose length of step is $\frac{1}{3}$ of a inch longer?

PAPER XXIV.

1. Multiply (i) 714·08 by ·005 ; (ii) 253·75 by 4·027.
2. Find the value of $\frac{3}{7}$ of 4s. 8d. + $3\frac{1}{2}$ of £2, 6s. − $\frac{6}{11}$ of £$5\frac{1}{2}$ + $\frac{1}{3}$ of a guine
3. If 483 yards of matting which is $1\frac{1}{4}$ yards broad are required to cover rectangular hall which is 26 yards 9 ins. long, what is the breadt of the hall?
4. In a country where goods have to be carried by men, and the averag man can carry $84\frac{1}{2}$ lbs., how many carriers will be required convey 8 tons 9 cwts. of goods?
5. If the water in a tank 8 feet long and 7 feet wide is 4 feet deep, and cubic foot of water weighs 1000 ozs., what is the weight in tons the water in the cistern?
6. Find the value of $\frac{5\frac{1}{7} - 3\frac{5}{14}}{2\frac{1}{2} + 3\frac{3}{4}} \times 79\frac{1}{2}$.
7. Express ·45895 lb. Troy in ozs. dwts. grs.

[Including Practice, Square Root, Unitary Method]

PAPER XXV.

1. The length and width of the bottom of a rectangular reservoir are 12 ft. 3 ins. and 8 ft. 4 ins. respectively; its contents are 38·28125 cu. yards. Find the depth in feet.
2. Find the value of £1·69375.
3. Divide 5·4 by 0·00072, and the quotient by 1470568 to four places of decimals.
4. What decimal of (i) 11 cwts. is 3 qrs. 21 lbs.?
   (ii) 10 kilograms is 75 grams?
5. Find the cost of an iron bridge weighing 216 tons 12 cwts. 2 qrs. at £32, 17s. 2d. per ton.
6. Find the square root of (i) 21893041; (ii) $8\frac{15}{49}$ to 3 places of decimals.
7. A cyclist takes 3 hours to ride 33 miles; find to the nearest yard his rate per minute.

PAPER XXVI.

1. What is the value of a block of certain material 5 ft. 3 ins. long, 2 ft. 4 ins. wide, and 1 ft. 2 ins. thick, worth 4 guineas a cubic foot?
2. Reduce £4, 18s. $10\frac{1}{2}$d. to the decimal of 5 guineas.
3. Find the value of 3·22887 of £1 correct to the nearest penny.
4. Find the value of 18 cwts. 1 qr. 21 lbs. at £27, 15s. per cwt.
5. When wheat is 15s. a bushel, 8 men can be fed for 12 days at a certain cost. For how many days can 6 men be fed for the same cost when wheat is 12s. a bushel?
6. Extract the square root of ·289 to 4 places of decimals.
7. What will be the weight in grams of 3 m. 75 cm. of cloth, if 10 m. of it weigh 2 Kg.?

PAPER XXVII.

1. Multiply (i) 98·003535 by ·0042074; (ii) 5·61023 by ·597001.
2. How many bricks 9 inches long, $4\frac{1}{2}$ inches wide, and 3 inches thick, ar required to make a wall 1 mile long, 7 feet high, and 18 inch thick?
3. Express £2, 17s. 4d. as the decimal of £13, 8s. 9d.
4. The circumference of a carriage-wheel is 2 yds. 2 ft. 2 ins.; how man times will it revolve during a journey of 27 mls. 578 yds. 2 f 10 ins.?
5. If a map is drawn to a scale of $\frac{1}{10000}$, what is the true area in sq. metr of a rectangular tract of land, which on the map is 9·5 cm. l 4·7 cm.
6. Find the square root of 5186·160225.
7. Find the cost of 1297 trucks of coal at £4, 17s. $11\frac{1}{2}$d. per truck.

PAPER XXVIII.

1. From $2\frac{5}{4}$ of a guinea take $\frac{7}{12}$ of a crown; and add $\frac{9}{16}$ of a shillin to $\frac{7}{32}$ of a sovereign.
2. A piece of metal which contains $\frac{1}{8}$ cu. inch weighs ·625 lbs. What the volume of a piece of the same metal which weighs 40 lbs.
3. Reduce £3, 12s. $2\frac{1}{4}$d. to the decimal of 11 guineas.
4. If 17 grams of ammonia gas occupy 22·2 litres, find, *correct to a decilitr* the volume occupied by 50 grams.
5. Find the cost of 94 lbs. 10 ozs. 18 dwts. 6 grs. at £3, 6s. 8d. per oz.
6. A box 4 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep (internal measurement weighs 10 lbs. If it be filled with water weighing 1000 ozs. f each cubic foot, what is the total weight of box and water?
7. Find the square root of $\frac{7}{5}$ to 4 places of decimals.

PAPER XXIX.

1. Find (to the nearest penny) the cost of 6 tons 7 cwts. 2 qrs. 17 lbs. at £3, 10s. 7d. a cwt.

2. A boy swims 3 miles in $5\frac{1}{2}$ hours; if he goes 2 ft. 8 ins. at each stroke, find how many strokes he takes per minute.

3. Divide ·01 by ·002, and 57·528 by 3·76.
How many times can a bottle which holds 0·125 gallon be filled from a cask containing 8·5 gallons?

4. How much will it cost to paint the walls of a room 19 ft. $10\frac{1}{4}$ ins. long, 16 ft. $1\frac{3}{4}$ ins. wide, and 10 ft. 3 ins. high, at $9\frac{1}{2}$d. per sq. yard?

5. How many times does a wheel, whose circumference is 3 yds. 2 ft. 7 ins., turn in rolling 1946 yards?

6. Find the square root of $34967\frac{2}{11}$ to 4 places of decimals.

7. A rate of 2s. $4\frac{1}{2}$d. in the pound is levied on a district whose rateable value is £369,782. What sum does this rate produce?

PAPER XXX.

1. Simplify (i) $\dfrac{4{\cdot}8 + {\cdot}009 - 1{\cdot}0017}{7{\cdot}4 \times {\cdot}065 \div {\cdot}13}$.

(ii) (·1268 ÷ ·4755) of 1s. 3d. − ·1875 × ·64 of 2s. 1d.

2. What is the cost of (i) 215 articles at £1, 15s. 7d. the score?
(ii) 5 Kg. 250 g. at £2, 6s. 8d. per Kg.?

3. If 1500 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of 25 sheets of the same size as the former?

4. How many yards of paper 20 ins. wide will be required for the walls of a room 24 ft. long, 18 ft. wide, and $12\frac{1}{2}$ ft. high, the doors and windows occupying 120 sq. feet?

5. On a map drawn to the scale of $\frac{1}{2500}$ the sides of an oblong field are 4·7 and 3·2 centimetres. Find the area of the field in sq. metres.

6. Divide 7·04 by ·0847 to four places of decimals.

7. Express 17 cwts. 3 qrs. 21 lbs. as the decimal of 1 ton.

PAPER XXXI.

1. Multiply 7·25 by 810·2, and divide $2\frac{1}{8} + \frac{5}{6} + 1\frac{11}{12}$ by $5\frac{1}{24} - 2\frac{23}{48}$.
2. Find the number of complete kilometres in 3040 miles, taking a centimetre to be 0·3937 ins.
3. On a map drawn to the scale of an inch to a mile, a certain estate covers an area of 2·44 sq. inches. Find the real area of the estate in acres.
4. If the wages of 7 men for 16 days are £18, 13s. 4d., what will be the wages of 6 men for 14 days?
5. What is the rent of 64 acres 3 roods 24 perches at £1, 6s. 8d. per acre?
6. Find the value of 1·46875 of £7, 4s.
7. An English standard of deal planks contains 120 planks 12 feet long, 9 inches wide, and 3 inches thick. How many cubic feet of wood does it contain?

PAPER XXXII.

1. Extract the square root of $6\frac{1}{7}$ to four decimal places.
2. Calculate the cost of repairing a road 25 m. 3 fur. 170 yds. long at £111, 16s. 8d. per mile.
3. Express in tons the weight of water on a rectangular field 420 yards long by 300 yards broad, when rain has fallen to a depth of 1 inch, supposing a cubic foot of water to weigh 62 lbs.
4. Assuming 10 acres to be equal to 404·7 ares, find, correct to the nearest integer, the number of acres in 500,000 sq. metres.
5. If a cistern measure 4·75 metres in length, 3·6 m. in breadth, and 1·94 m. in height, find how many litres it will hold.
6. (i) Multiply, and (ii) divide, 169·3034 by ·00045.
7. If the weight of 5 sovereigns is 41 grams, how many sovereigns will weigh 2 Kg. 50 g.?

[Including Mensuration of Circle and Cylinder, Ratio, Percentages, Profit and Loss]

PAPER XXXIII.

1. Find the cost, *to the nearest penny*, of 105 boxes of tea, each containing 3 qrs. 10 lbs. 8 ozs. at £12, 16s. 8d. per cwt.

2. A motor car travels for the first 20 minutes at the rate of 6 Km. 126 m. per hour, then for 2 hours 20 minutes at the rate of 30 Km. per hour, and afterwards for 25 minutes at the rate of 7 Km. 152 m. per hour ; find, in kilometres, the total distance traversed by the car.

3. The tonnage of a vessel is $\frac{(L-\frac{3}{5}B)\times B\times \frac{1}{2}B}{94}$ where L is the length of keel between perpendiculars in feet and B the breadth of the vessel in feet. What is the tonnage of a vessel for which L is 287 feet and B 32 feet ?

4. If the whole revenue of a country, £50,000,000, were paid as interest on the National Debt, £760,000,000, how much per cent. would it give ?

5. By selling an article for £9, 5s. 0d. a person loses 5 per cent. ; how much per cent. would he gain by selling it for £11, 6s. 0d. ?

6. Find, *to the nearest pound*, the weight of a 5-mile length of copper wire 0·03 inches in radius. The weight of a cubic foot of copper is 560 lbs. ($\pi = 3{\cdot}14$.)

7. If the wages of 11 labourers for 15 weeks be £103, 2s. 6d., in what time will the wages of 13 labourers amount to £170, 12s. 6d. at the same rate per week ?

PAPER XXXIV.

1. If a postage stamp measures $\frac{11}{12}$ of an inch by $\frac{13}{16}$ of an inch, how many stamps will be required to cover a wall 16 ft. 3 ins. by 13 ft 9 ins. ?

2. In 1900-1901 the national expenditure was £183,592,264 ; of this the army cost £91,710,000, the navy £29,520,000, and the civil service £23,500,000. Express each of these as a percentage of the whole *to the nearest integer.*

3. If 8 men would take 75 days to complete a piece of work, how many men would be required to finish it in 40 days ?

4. (i) Find the number that is to $7\frac{2}{3}$ in the ratio of £3, 1s. 3d. to £4, 13s. 11d.

   (ii) Divide 204 into three parts proportional to 7, 8, 9.

5. A builder sold a house for £945, thereby gaining 8 per cent. on his outlay ; what did it cost him to build ?

6. Divide (i) ·00230568 by ·0312 ; (ii) ·07287 by ·0063.

7. How many times will a bicycle wheel, 12 ft. in circumference, revolve between two places distant 12 mls. 1 fur. from each other ?

PAPER XXXV.

1. The annual divisible receipts of a railway company are £437,500 and there are 250,000 shares at £21 each ; what would be the dividend for each share, and what rate per cent. would be paid on each share ?

2. Express 25 acres as the fraction of $1\frac{1}{2}$ sq. miles, and find the value of ·06875 of £20.

3. What length of paper, 20 inches wide, will it take to cover the walls of a room 16 ft. long, 12 ft. 3 ins. wide, and 10 ft. high ?

4. If 4 men plough 72 acres in 9 days, how long will it take 5 men to plough 10 acres ?

5. Each metre of a cable weighs 2·078 Kg. Find to the nearest metre the length of cable which weighs 1000 Kg.

6. Express in their simplest forms the ratio of—
   (i) $2\frac{1}{4}$ to $7\frac{1}{3}$.
   (ii) $\frac{3}{4}$ of 53 cwts. 3 qrs. 3 lbs. to 0·4 of 65 cwts. 11 lbs.

7. By selling an article for 38s. the seller loses 5 per cent.; what must he sell it for to gain 10 per cent.?

PAPER XXXVI.

1. Find the cost of painting the four sides and bottom of a tank $2\frac{1}{2}$ yds. long, 4 ft. wide, and $4\frac{1}{2}$ ft. deep, at 4d. per sq. ft.

2. The Colonial Office grant for 1900-1 was £55,250, and the salary for the principal secretary £5000; what percentage of the whole was this? Answer *to the nearest integer.*

3. (i) Express ·0390625 of a day in minutes and seconds.
   (ii) If 8 metres are equal to 315 inches, express 1 metre as a decimal of a yard.

4. Find the H.C.F. and L.C.M. of 336, 468, 792.

5. Reduce £3, 2s. 3d. to the decimal of £6.

6. By selling a horse for £85 a man lost $7\frac{1}{2}$ per cent.; find what would have been his gain or loss per cent. if it had been sold for £96, 10s. 0d.

7. How many lengths of 12·45 centimetres each can be taken from a strip of cloth 3·27 metres long, and what will be the length of the remaining piece?

PAPER XXXVII.

1. An army of 145,600 men loses $12\frac{1}{2}$ per cent. of its number through disease, and 15 per cent. of the remainder in battle; how many men are left?

2. A picture, 18 inches broad and $10\frac{1}{2}$ inches deep, is to be photographed on a reduced scale, so that the breadth of the photograph may be $7\frac{1}{2}$ inches. Find the depth of the photograph.

3. Divide (i) 23·78 by 62·5 ; (ii) 6400 by ·04096.
4. If the circumference of a coach wheel measures 17 ft. $7\frac{1}{5}$ ins., how of will it turn round in traversing a distance of 8 miles 264 feet ?
5. By selling tea at 4s. 6d. per lb., a grocer cleared one-eleventh of outlay ; what would he have gained if he had sold the tea at 5s. ?
6. By selling a horse for £66 I should lose 28 per cent. of the cost at wh I bought the animal ; what is my loss ?
7. If a gallon contains 277·29 cu. inches, how many gallons will be contai in a rectangular cistern, whose length, breadth, and depth are 13 7 ft. 6 ins., and 6 ft. 7 in. ?

PAPER XXXVIII.

1. Divide 195 by 0·00208 and 0·208 by 19500.
2. In a school of 1200 the number presented for an examination was 20 cent. of the whole, while the numbers passing in the first and sec classes were respectively $12\frac{1}{2}$ per cent. and $32\frac{1}{2}$ per cent. of th presented. Find the number presented and the number passing each class.
3. How many planks, each $13\frac{1}{2}$ ft. long and $10\frac{1}{2}$ ins. wide, will be requi for the construction of a platform 54 yards long and 21 yards bro What will be the cost at $5\frac{1}{2}$d. per sq. ft. ?
4. The Education vote for 1900-1901 was £12,563,455, and of £8,511,170 was allotted to elementary schools ; what percentag the whole was this ? (Answer to the *nearest integer*.)
5. If 7 oxen can eat 3 tons of hay in a month, how long will it take sheep to eat 35 tons, an ox consuming three times as much sheep ?
6. A, B, and C enter into partnership and supply respectively £300, £ and £450 as capital. At the end of the year the profits amoun £450. Find, *to the nearest penny*, the share of each person.
7. A watch is offered for sale at £5, 15s. 0d., and, if that price is redu by 5 per cent., the dealer who is selling it will still make $9\frac{1}{4}$ per c profit ; how much did the watch cost him ?

PAPER XXXIX.

1. On a road 130 kilometres long, 26 kilometres are flat. How much per cent. is this?
2. Express as a decimal $1 - 6 \times (0{\cdot}9) + 15 \times (0{\cdot}9)^2 - 20 \times (0{\cdot}9)^3 + 15 \times (0{\cdot}9)^4 - 6 \times (0{\cdot}9)^5 + (0{\cdot}9)^6$.
3. 10 tons of goods bought at 27s. 6d. per cwt. are sold at £30 per ton. What is the whole gain, and the gain per cent.?
4. Find the value of ·0625 of 1 ton 2 cwts. 3 qrs. 12 lbs.
5. Extract the square root of 108241.
6. The National Income in 1893 was £98,297,000, and in 1902 £161,199,000. At what rate per cent. was this increase? (Answer *to the nearest integer.*)
7. A room is 21 ft. long, 14 ft. wide, and 12 ft. high; what length of paper, 27 ins. wide, would be required to paper its four walls and its ceiling?

PAPER XL.

1. Divide £157, 10s. 0d. among three persons so that the first may have twice as much as the second, and the third half as much as the first and second together.
2. If 81 gallons of water will fill a cistern 4 ft. 4 ins. long, 2 ft. 8 ins. broad, and 1 ft. $1\frac{1}{2}$ ins. deep, how many cu. inches are contained in a pint?
3. Divide (i) ·001036656 by 50·08; (ii) 216·711 by ·008756.
4. Out of a revenue of £143,581,500 for 1901-2, the tax on tea came to £5,793,400; what percentage (to the nearest integer) was this?
5. To reap 210 acres of corn 20 men are employed for 10 days, working 14 hours a day. How many acres will 27 men reap in 15 days, working 12 hours a day?
6. If the manufacturer makes a profit of 25 per cent., the agent one of 8 per cent., and the shopkeeper one of 20 per cent., what is the cost to the manufacturer of an article which is sold in the shop for £32, 8s. 0d.?
7. Find, in sq. decimetres, the area of a circle whose radius is 5·1 metres. ($\pi = 3{\cdot}1416$.)

[Including Simple Interest]

PAPER XLI.

1. A room is 18 ft. long, 12 ft. wide, and 11 ft. high; what length paper, a yard wide, would be required to cover its four walls a ceiling?

2. A bankrupt's estate amounts to £910, 3s. 1½d., and his debts to £18 What can he pay in the pound?

3. Out of the 'Breakfast Table' taxes of £12,852,000 for 1901 £6,399,000 were for sugar. What percentage (to the near integer) of the whole was this?

4. At what rate of simple interest would £1510 amount to £1691, 4s. in three years?

5. Express 17 lbs. 10 ozs. 6 dwts. 15 grs. as a decimal of 1 lb. Troy; a also as a decimal of 1 lb. Avoirdupois.

6. Find the number of years in which £500 amounts to £561, 17s. 6d. 1½ per cent.

7. Three partners invest £3000, £1200, and £2000 respectively in a comm business, and at the end of the year the profits are £744; what w be the share of each partner?

PAPER XLII.

1. How many equal sums of £1, 17s. 7½d. can be given away out of a s of £208, 16s. 9d., and what sum will be left over?

2. Find *to the nearest centimetre* the depth of water in a cistern 3·27 met long, 2·35 metres wide, which holds 7000 litres.

3. At what rate per cent. will £126, 13s. 4½d. double itself in 30 years?

4. Find the expense *to the nearest penny* of covering with paper 1 ft. 9 ins. wide, at $3\frac{1}{2}$d. per yd., the walls of a room 25 ft. long, 23 ft. $7\frac{1}{2}$ ins. wide, and 16 ft. high.
5. The revenue from Customs was £19,707,000 in 1893, and in 1902 was £34,443,000. Find (*correct to* 1 *place of decimals*) the rate per cent. increase.
6. Find the total rental of a town, if a tax of 9d. in the pound on rent brings in £2163, 14s. 3d.
7. Find the square root of 0·03456789 to 5 decimal places.

PAPER XLIII.

1. How many yards of carpet, $\frac{3}{4}$ yd. wide, will cover the floor of a room whose width is 20 feet and length 22 feet?
2. At what rate must simple interest be charged in order that £2375 may increase to £2826, 5s. 0d. in 4 years?
3. Find the square root of 5·2 to three decimal places.
4. Find the amount of £325, 13s. 4d. at 3 per cent. for 4 years at simple interest. (Answer to the nearest penny.)
5. If 27 men, working $8\frac{1}{2}$ hours a day, can finish a certain piece of work in 16 days, in how many days can 68 men, working 9 hours a day, do the same work?
6. A merchant buys a 50-gallon cask of wine for £62, 10s. 0d., and sells it at 26s. per gallon; what is his gain per cent.?
7. If you invest a sum of money in such ways that on one-third of it you gain 3 per cent., on one-fifth you lose 4 per cent., and on the remainder gain 6 per cent., what rate per cent. do you make on the whole sum invested?

PAPER XLIV.

1. A Turkey carpet, measuring 12 ft. 6 ins. by 11 ft. 6 ins., is laid down on the floor of a room measuring 14 ft. by 13 ft. Determine the quantity of floorcloth necessary to complete the covering of the area, and its price at 4s. per sq. yd.
2. The cost of 5 cwts. 2 qrs. 14 lbs. of coffee is £56, 13s. 9d. If the whole is sold at 1s. $11\frac{1}{2}$d. per lb., what is the total gain or loss?

3. A sum of money put out at simple interest amounts to £688, when th rate is $2\frac{1}{2}$ per cent. and the time 3 years. What would the amoun be if the rate were $3\frac{1}{4}$ per cent. and the time $2\frac{1}{3}$ years?

4. Given that the volume of a mass of gas is inversely proportional to th pressure, and that it measures 25·5 cu. dms. when the pressure i 750 mm. of mercury, what will be its volume when the pressure i 450 mm. of mercury?

5. A sum of £237, 18s. 4d., lent at simple interest, amounts in 3 years t £270, 0s. $8\frac{1}{2}$d. What is the rate per cent.?

6. In 1880 the whole trade between Germany and Canada was £110,000 while in 1902 it was worth £2,703,000; find, correct to 1 *place o decimals*, the rate per cent. increase.

7. A rectangular area, the length being five times the breadth, contain 70,000 acres. Find (to the nearest mile) the amount of fencin required to enclose it.

[Including Contracted Work, Compound Interest, Stocks, Discount]

PAPER XLV.

1. Multiply 32·3475 by 16·4321 correct to 3 places of decimals.
2. What will £1000 amount to in 3 years at 5 per cent. compound interest
3. A square field contains 6 ac. 2 rds. 581 sq. yds.; find the length of on side in yards.
4. A cistern which is 9 ft. 4 in. long and 7 ft. 6 in. wide contains 6 ton 5 cwts. of water; if a cubic foot of water weigh 1000 ozs., what i the depth of water in the cistern?
5. Find the value of $\frac{1}{\sqrt{19}}$ to 4 places of decimals.
6. By selling 12 lbs. of tea for 31s. 6d. I gain 5 per cent.; what do I gai or lose per cent. by selling 50 lbs. of the same tea for £6, 9s. 2d.?
7. Divide 875·42137 by 22·0157 correct to 3 places of decimals.

PAPER XLVI.

1. If 1 gram contains 15·432 grains, find the number of kilograms in 1 cwt., correct to 2 places of decimals.
2. Find, *correct to the nearest penny*, the compound interest on £1272 for 3 years at $4\frac{3}{4}$ per cent. per annum.
3. Find the true discount on £793, 14s. $10\frac{1}{2}$d. due 9 months hence at 4 per cent.
4. Given that the area of the surface of a sphere is $4\pi r^2$, where $\pi = 3{\cdot}142$, and r is the radius of the sphere; find the area (in sq. decimetres *correct to* 3 *decimal places*) of the surface of a sphere whose radius is 7·2 dms.
5. What income would be obtained by investing £12002, 12s. $4\frac{1}{2}$d. in 3 per cent. stock at 93?
6. Find the value of $\frac{1}{7}+\frac{1}{7.9}+\frac{1}{7.9.11}+\frac{1}{7.9.11.13}+\ldots\ldots$ so that the error may be not greater than ·00005.
7. Find the area in sq. dekametres of a path 1·5 metres broad running round the edges of a rectangular garden, which is 80·52 metres long and 33·45 metres broad.

PAPER XLVII.

1. Find the square roots of ·00255025 and 53111·8116.
2. Find, *correct to the nearest penny*, the difference between the simple and compound interest on £2343, 15s. 0d. for 2 years at 4 per cent. per annum.
3. Find the present worth of a bill of £1013, 2s. 6d. due $5\frac{1}{4}$ months hence, and discounted at 3 per cent. per annum.
4. A person invests £1730 by placing £900 of it in $2\frac{3}{4}$ per cent. stock at $112\frac{3}{8}$, and the remainder in 5 per cent. stock at $103\frac{5}{8}$. Find, to the nearest penny, the average rate of interest he receives on his money.
5. Multiply 5·61177 by 0·71279 correct to 5 places of decimals.
6. If iron weighs 7·8 times the same volume of water, find the length of a bar of iron 2 ft. 6 ins. by 1·7 ft. which weighs 10 tons. (1 cu. ft. of water weighs 1000 ozs.) Give the answer correct to one-tenth of an inch.
7. How many wooden blocks, $9\frac{1}{2}$ ins. long and $4\frac{1}{4}$ ins. wide, would be required to pave a floor measuring 25 ft. 4 ins. by 19 ft. $1\frac{1}{2}$ ins.?

PAPER XLVIII.

1. A rectangular tank is 18 ft. long, 14 ft. broad, and 10 ft. deep. Find the cost of painting the sides and bottom of the inside at 1s. $1\frac{1}{2}$d the sq. yd.
2. Simplify $\dfrac{6\frac{1}{2}-3\frac{1}{4}}{7\frac{1}{3}+1\frac{4}{7}} \div \dfrac{4\frac{1}{3} \text{ of } 1\frac{1}{2}}{2\frac{3}{7} \text{ of } 3\frac{3}{5}}$.
3. Find the length of the side of a square plot of land $2\frac{1}{2}$ acres in extent (Answer in yards.)
4. Find the present worth of £504, 15s. 0d. due 73 days hence at $4\frac{3}{4}$ per cent.
5. Find, to the nearest penny, the compound interest on £3000 for 3 years at $2\frac{1}{4}$ per cent. per annum.
6. Find the value of $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+\ldots\ldots$ correct to 3 places of decimals.
7. A man invests a fourth of his capital in $2\frac{1}{4}$ per cent. stock at 90, and the remainder in $3\frac{1}{2}$ per cent. stock at 105; find the average rate per cent. return on his capital.

PAPER XLIX.

1. Find the present worth of £84, 10s. 0d. due 2 years hence, compound interest being reckoned at 4 per cent. per annum.
2. A square plot of ground is sold for building purposes at 6d. per sq. foot and realises £2059, 4s. 6d. What is the length, in feet, of each side of the plot?
3. The National Debt increased from £635,040,965 in 1899 to £800,443,386 in 1903. Find (correct to 1 place of decimals) the rate per cent. at which it increased.
4. A rectangular garden 50·5 metres long, and 29·7 metres broad, consists of a rectangular grass plot surrounded by a path 1·2 metres broad. Find (in sq. metres) the area of the path.
5. A plot of land is sold at £1200 per acre; what is the price in francs per sq. metre, given that £1 = 25 francs and 1 metre = 39·375 inches? Answer correct to 2 places of decimals.
6. Multiply 42·36068 by ·0236068 correct to 6 places of decimals.
7. What is the gain by investing £1950 at $97\frac{3}{8}$ and selling out at 104, paying brokerage on each transaction at $\frac{1}{8}$ per cent.?

[Miscellaneous]

PAPER L.

1. Find the number of sq. metres of wood, 1·5 cm. thick, required for the construction of a closed rectangular box, the external dimensions of which are 3·5 metres, 2·4 metres, and 1·3 metres.

2. If £3 are worth 20 thalers, 25 thalers worth 93 francs, 25 francs worth 5 scudi, and 62 scudi worth 135 gulden, how many gulden are equivalent to £1?

3. Find the value of $\frac{1}{\sqrt{23}}$ to 5 places of decimals.

4. What will be the gain per cent. if eggs, bought at the rate of 6 for 5d., are sold at the rate of 5 for 6d.?

5. The time (in seconds) of a complete oscillation of a pendulum is obtained from the expression $2\pi\sqrt{\frac{l}{g}}$, where $\pi = 3{\cdot}142$, $g = 32$, and l = length of pendulum in feet. Find in seconds, *correct to* 1 *place of decimals*, the time of oscillation of a pendulum 58 ft. long.

6. Multiply 1·231056 by ·81231056 *correct to* 7 *places of decimals.*

7. When the 3 per cents. are at $81\frac{1}{2}$ and the $3\frac{1}{2}$ per cents. at $95\frac{1}{4}$, for what sums respectively can I purchase £1725 in each stock?

PAPER LI.

1. A and B have together £2, 18s. 0d.; B and C have together £1, 18s. 0d.; C and A have together £1, 11s. 0d. How much has each separately?

2. An article is first sold at a profit of 10 per cent. The purchaser again sells it for 4s. 7d., and calculates that his gain is 15 per cent. of the price at which he sells it. How much did the article originally cost?

3. If either 5 men or 9 boys can do a certain piece of work in 19 days, in how many days can 13 men and 7 boys, working together, do a piece of work twice as great?

4. Find (in sq. metres) the total area of the surface of a rectangular box 17·5 dm. long, 14·6 dm. broad, and 9·7 dm. deep.

5. One pipe fills a cistern in 12 minutes, a second in 15 minutes, and a third empties it in 20 minutes. The first two are opened until the cistern is one-third full, and then the third pipe is opened also. How long will it be from the start before the cistern is full?

6. Find the value of $13{\cdot}2547 \times {\cdot}0921 \times 823{\cdot}14$, *correct to* 3 *places of decimals.*

7. Find the value of $\frac{1}{\sqrt{29}}$ to 4 places of decimals.

PAPER LII.

1. Find the number of bricks required to build a wall 26 yards long, 7 ft. 8 ins. wide, and 14 ins. thick, reckoning for each brick and its mortar a length of 9·1 inches, a breadth of 4·6 inches, and a depth of 3·2 inches.

2. Multiply 72·587032 by 0·078953 correct to 4 places of decimals.

3. The total area of three estates is 1768 acres. If the areas of the two smaller estates be respectively three-fifths and two-thirds that of the largest, find the acreage of each.

4. Find the square root of ·98765321 to 4 places of decimals.

5. A centimetre being ·3937 inches, find to four significant figures how many acres are in a square kilometre.

6. £5280 are invested partly in the 3 per cents. at 102 and partly in the $2\frac{1}{2}$ per cents. at 91, so that the income derived from each source is the same. What is the income produced from the total investment?

7. Find the breadth of a room 21 ft. 8 in. long, if the cost of covering the floor with carpet 27 ins. wide, at 3s. $4\frac{1}{2}$d. per yard, is £8, 18s. 9d.

PAPER LIII.

1. The weekly wages of a number of bricklayers amount to £130, 12s. 6d., the rate of paying being 9½d. an hour; to what will they amount if the rate is raised to 10d. an hour?
2. Divide £100 between 3 men, 5 women, 4 boys, and 3 girls, so that a man receives as much as 5 girls, a woman as much as 4 girls, and a boy as much as 3 girls.
3. How many francs are equivalent to £43, 15s. 0d. at the rate of 25·2 francs in a sovereign?
4. Air contains 23·01 per cent. of its weight of oxygen. If a cubic foot of air weigh 1·23 ounces, what will be the number of ounces (correct to 2 places of decimals) of oxygen in a room 18 feet by 27 ft. 4 ins. by 12 ft.?
5. Divide 57·43289 by 71·0134872 correct to 4 places of decimals.
6. A piece of copper, weighing 3 lbs. 2 ozs., is dropped gently into a vessel full of water. Find (correct to $\frac{1}{100}$ of a cubic inch) the volume of water that will be displaced, assuming that a cubic foot of water weighs 1000 ozs., and that copper is 8·915 times heavier than water.
7. The length of a rectangular piece of ground is twice its breadth; its area is 30479·805 sq. ft. Find its length and breadth.

PAPER LIV.

1. A tank contains 9100 litres, and another tank of the same horizontal section, but 103 centimetres deeper, has a capacity of 10750 litres. Find the area of the section in sq. metres and the depth of the first tank in metres. (Answer correctly to the second decimal place.)
2. If water is 1·0841 times as heavy as oak, and 1 cu. cm. of water weighs 1 gram, find in kilograms the weight of 2051 planks of oak, each 3 metres 5 centimetres long, 22 centimetres wide, and 7 centimetres 4 millimetres thick.

3. On March 22 the sun rises at 6 o'clock and sets at 6.14; on April 22 it rises at 4.52 and sets at 7.6. What is the average daily lengthening of daylight for this period? Give your answer to the nearest second.

4. Two clocks, of which one loses 1 min. 24 sec. in the day, and the other gains 44 seconds in the day, are put right at noon on March 16. When will they again indicate the same time, and what time will they then show?

5. How long (in minutes) will it take to describe the boundary of a square field containing 13 ac. 81 sq. yds., at the rate of $3\frac{3}{4}$ mls. an hour.

6. Assuming that the action of a river in a cañon wears down the rock at the rate of a hundredth of an inch in a year, how long has it taken to make a cañon 1500 feet deep?

7. In a distilling operation, during 5 hours the fluid contained 62·5 per cent. of alcohol, during $4\frac{1}{2}$ hours 31·4 per cent., and during the remaining 3 hours 18·4 per cent. What is the average strength of the whole fluid distilled over, assuming that it came over at a uniform rate during the whole time?

PAPER LV.

1. The weight of a cubic inch of water is 253·17 grains, that of a cubic inch of air ·31 grains. Find to three places of decimals how many cubic inches of water are equal in weight to one cubic foot of air.

2. A person on the platform of a station, which is 132 yards long, notices that a train, which takes three seconds to pass him, occupies $7\frac{1}{2}$ seconds in passing completely through the station. What is the speed of the train in miles per hour?

3. Goods are imported from abroad at an expense equal to 35 per cent. of the cost of production; and the importer makes 15 per cent. on his whole outlay by selling them to a tradesman at £7, 15s. 3d. per ton. Find the cost per ton of production.

4. The basin of the Severn is 4350 sq. miles in area, and the yearly rainfall 29 inches. Half of the rain that falls is again evaporated, and the rest flows to the sea. Find, to the nearest million gallons, how much water flows to the sea per hour. (1 cu. ft. of water occupies 6·23 gallons.)

5. From observation of the eclipses of Jupiter's satellites, it is found that the time taken by light to travel from the sun to the earth is 499 seconds (with a possible error of 2 seconds either way). If the velocity of light is 186,330 miles per second, find the distance from the sun to the earth, and the possible error in your result. (Answer to the nearest 100,000 miles.)

6. A cubical block of metal of 7·84 inches side weighs ·25 lb. per cubic inch. A hole of square sectional area is to be cut completely through the metal perpendicular to a face of the cube, in order that the weight of metal left may be 100 lbs. Find, correct to 2 places of decimals, the side of the square section.

7. A hollow ball, of external diameter 6 centimetres and of thickness $\frac{1}{2}$ centimetre, weighs 162 grams when it is filled with liquid. Find, *to the nearest hundredth of a gram*, the weight of a cubic centimetre of the liquid, a cubic centimetre of the substance of the ball weighing 2 grams. (If a sphere is inscribed in a cube, then volume of sphere $= \frac{11}{21}$ volume of cube.)

$$P = \frac{100 \times S.J.}{R \times T}$$

$$R = \frac{100 \times S.J.}{P \times T}$$

$$T = \frac{100 \times S.J.}{P \times R}$$